

# Recurrent Neural Networks – *under the hood*

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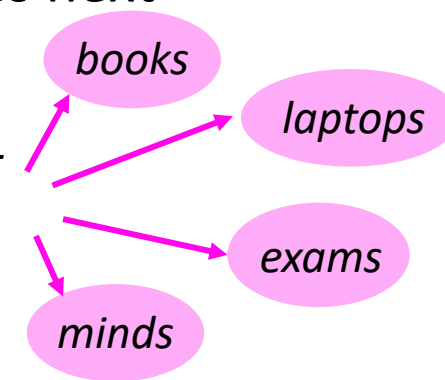
CS 114B

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## 2. Language Modeling

- **Language Modeling** is the task of predicting what word comes next

*the students opened their \_\_\_\_\_*



- More formally: given a sequence of words  $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(t)}$ , compute the probability distribution of the next word  $\mathbf{x}^{(t+1)}$ :

$$P(\mathbf{x}^{(t+1)} \mid \mathbf{x}^{(t)}, \dots, \mathbf{x}^{(1)})$$

where  $\mathbf{x}^{(t+1)}$  can be any word in the vocabulary  $V = \{\mathbf{w}_1, \dots, \mathbf{w}_{|V|}\}$

- A system that does this is called a **Language Model**

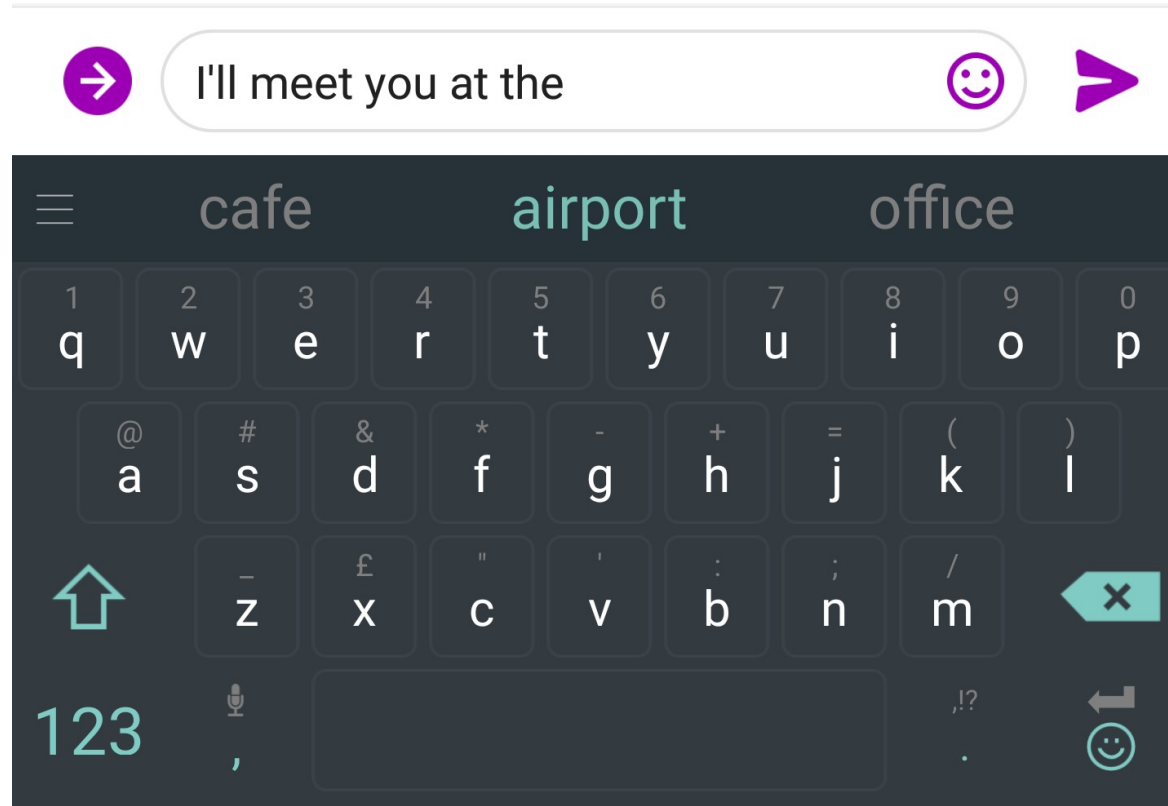
# Language Modeling

- You can also think of a Language Model as a system that assigns a probability to a piece of text
- For example, if we have some text  $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(T)}$ , then the probability of this text (according to the Language Model) is:

$$\begin{aligned} P(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(T)}) &= P(\mathbf{x}^{(1)}) \times P(\mathbf{x}^{(2)} | \mathbf{x}^{(1)}) \times \dots \times P(\mathbf{x}^{(T)} | \mathbf{x}^{(T-1)}, \dots, \mathbf{x}^{(1)}) \\ &= \prod_{t=1}^T P(\mathbf{x}^{(t)} | \mathbf{x}^{(t-1)}, \dots, \mathbf{x}^{(1)}) \end{aligned}$$


This is what our LM provides

# You use Language Models every day!



# You use Language Models every day!



what is the | 

what is the **weather**  
what is the **meaning of life**  
what is the **dark web**  
what is the **xfi**  
what is the **doomsday clock**  
what is the **weather today**  
what is the **keto diet**  
what is the **american dream**  
what is the **speed of light**  
what is the **bill of rights**

[Google Search](#) [I'm Feeling Lucky](#)

# n-gram Language Models

*the students opened their \_\_\_\_\_*

- **Question:** How to learn a Language Model?
- **Answer** (pre- Deep Learning): learn an *n*-gram Language Model!
- **Definition:** An *n*-gram is a chunk of  $n$  consecutive words.
  - **unigrams:** “the”, “students”, “opened”, “their”
  - **bigrams:** “the students”, “students opened”, “opened their”
  - **trigrams:** “the students opened”, “students opened their”
  - **four-grams:** “the students opened their”
- **Idea:** Collect statistics about how frequent different n-grams are and use these to predict next word.

# n-gram Language Models

- First we make a **Markov assumption**:  $x^{(t+1)}$  depends only on the preceding  $n-1$  words

$$P(\mathbf{x}^{(t+1)} | \mathbf{x}^{(t)}, \dots, \mathbf{x}^{(1)}) = P(\mathbf{x}^{(t+1)} | \overbrace{\mathbf{x}^{(t)}, \dots, \mathbf{x}^{(t-n+2)}}^{n-1 \text{ words}}) \quad (\text{assumption})$$

prob of a n-gram  $\rightarrow$

$$= \boxed{P(\mathbf{x}^{(t+1)}, \mathbf{x}^{(t)}, \dots, \mathbf{x}^{(t-n+2)})}$$

prob of a (n-1)-gram  $\rightarrow$

$$\boxed{P(\mathbf{x}^{(t)}, \dots, \mathbf{x}^{(t-n+2)})}$$

(definition of conditional prob)

- Question:** How do we get these  $n$ -gram and  $(n-1)$ -gram probabilities?
- Answer:** By **counting** them in some large corpus of text!

$$\approx \frac{\text{count}(\mathbf{x}^{(t+1)}, \mathbf{x}^{(t)}, \dots, \mathbf{x}^{(t-n+2)})}{\text{count}(\mathbf{x}^{(t)}, \dots, \mathbf{x}^{(t-n+2)})} \quad (\text{statistical approximation})$$

# n-gram Language Models: Example

Suppose we are learning a 4-gram Language Model.

~~as the proctor started the clock, the~~ *students opened their* \_\_\_\_\_  
discard condition on this

$$P(\mathbf{w} | \text{students opened their}) = \frac{\text{count}(\text{students opened their } \mathbf{w})}{\text{count}(\text{students opened their})}$$

For example, suppose that in the corpus:

- “students opened their” occurred 1000 times
  - “students opened their *books*” occurred 400 times
    - $\rightarrow P(\text{books} | \text{students opened their}) = 0.4$
  - “students opened their *exams*” occurred 100 times
    - $\rightarrow P(\text{exams} | \text{students opened their}) = 0.1$
- Should we have discarded the “proctor” context?



# Sparsity Problems with n-gram Language Models

## Sparsity Problem 1

**Problem:** What if “students opened their  $w$ ” never occurred in data? Then  $w$  has probability 0!

**(Partial) Solution:** Add small  $\delta$  to the count for every  $w \in V$ . This is called *smoothing*.

$$P(w | \text{students opened their}) = \frac{\text{count}(\text{students opened their } w)}{\text{count}(\text{students opened their})}$$

## Sparsity Problem 2

**Problem:** What if “students opened their” never occurred in data? Then we can’t calculate probability for *any*  $w$ !

**(Partial) Solution:** Just condition on “opened their” instead. This is called *backoff*.

**Note:** Increasing  $n$  makes sparsity problems worse. Typically, we can’t have  $n$  bigger than 5.

# Storage Problems with $n$ -gram Language Models

**Storage**: Need to store count for all  $n$ -grams you saw in the corpus.

$$P(\mathbf{w} | \text{students opened their}) = \frac{\text{count}(\text{students opened their } \mathbf{w})}{\text{count}(\text{students opened their})}$$

Increasing  $n$  or increasing corpus increases model size!

# n-gram Language Models in practice

- You can build a simple trigram Language Model over a 1.7 million word corpus (Reuters) in a few seconds on your laptop\*

Business and financial news

today the \_\_\_\_\_

get probability  
distribution

company	0.153
bank	0.153
price	0.077
italian	0.039
emirate	0.039
...	

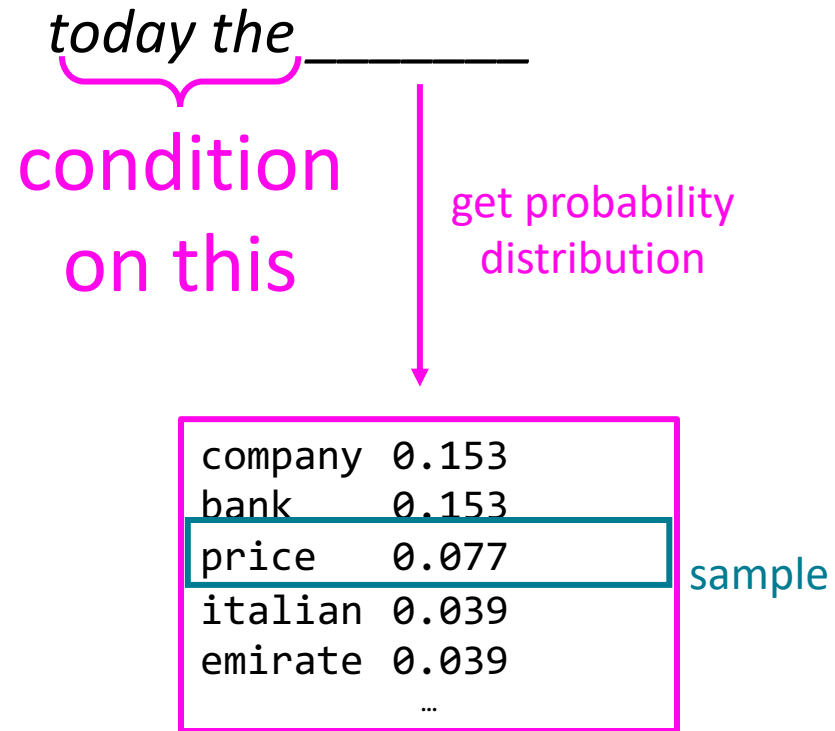
**Sparsity problem:**  
not much granularity  
in the probability  
distribution

Otherwise, seems reasonable!

\* Try for yourself: <https://nlpforhackers.io/language-models/>

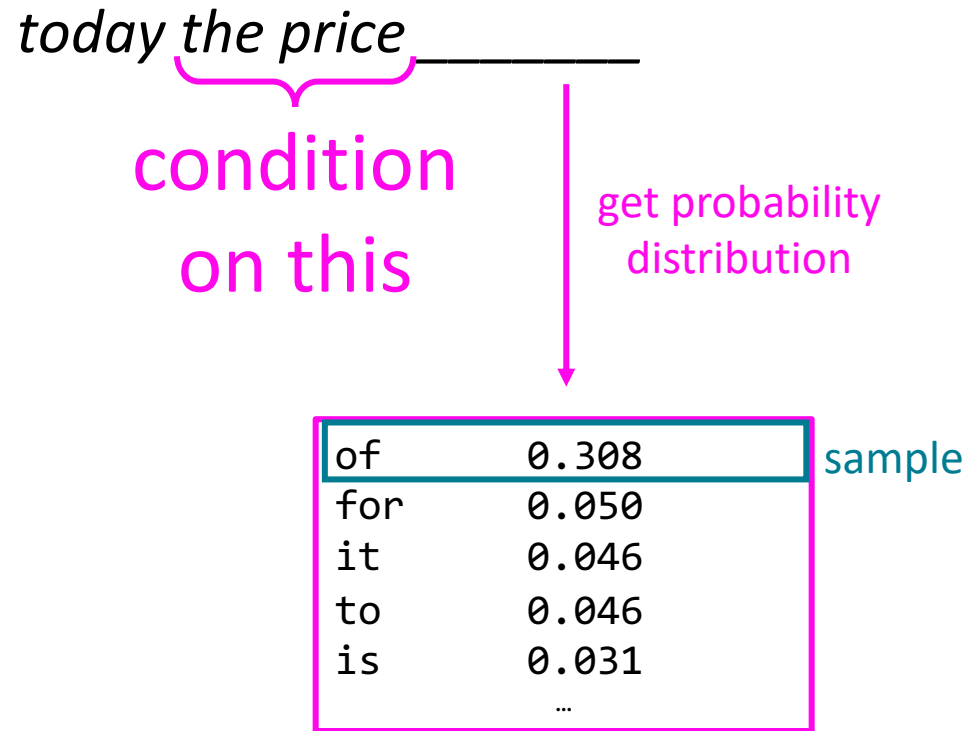
# Generating text with a n-gram Language Model

You can also use a Language Model to generate text



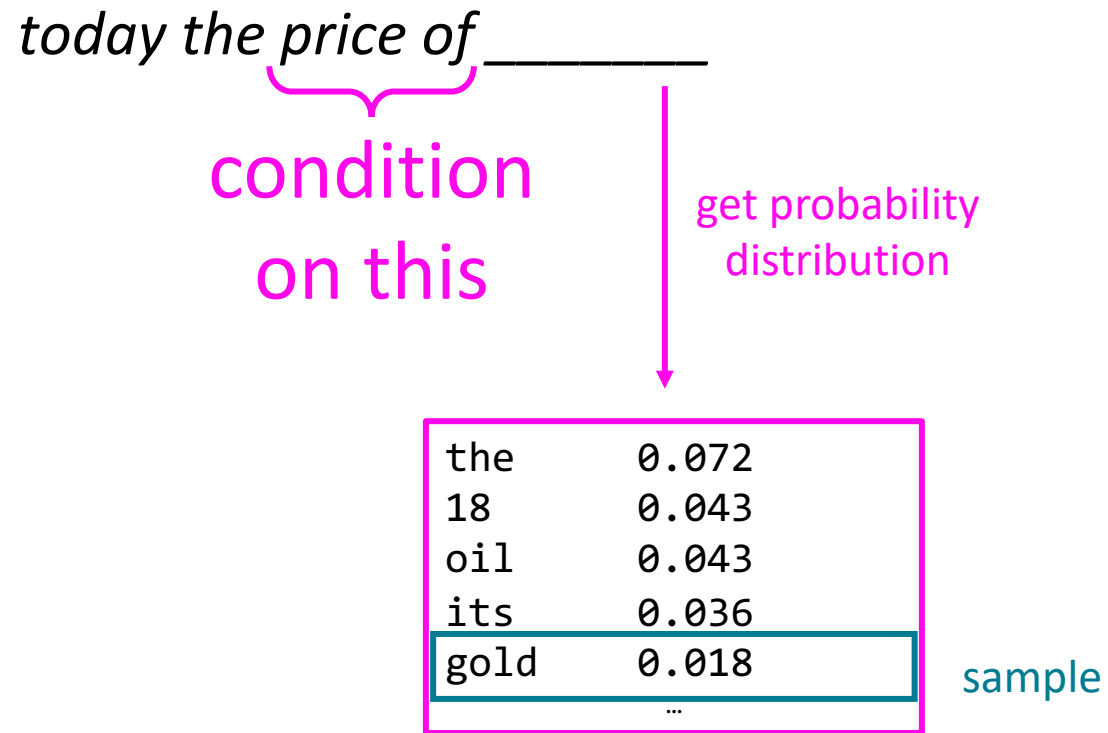
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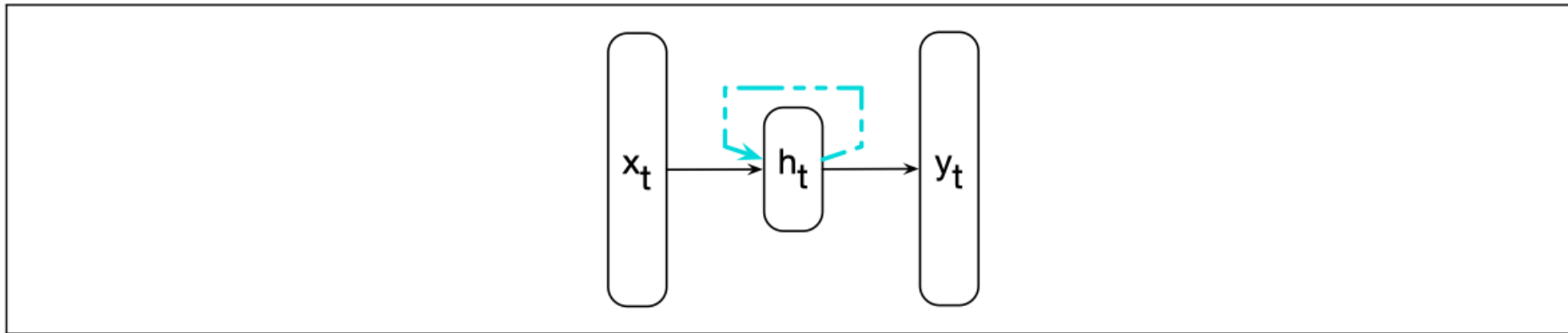
*today the price of gold per ton , while production of shoe lasts and shoe industry , the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .*

Surprisingly grammatical!

...but **incoherent**. We need to consider more than three words at a time if we want to model language well.

But increasing  $n$  worsens sparsity problem,  
and increases model size...

# Elman's Simple RNN



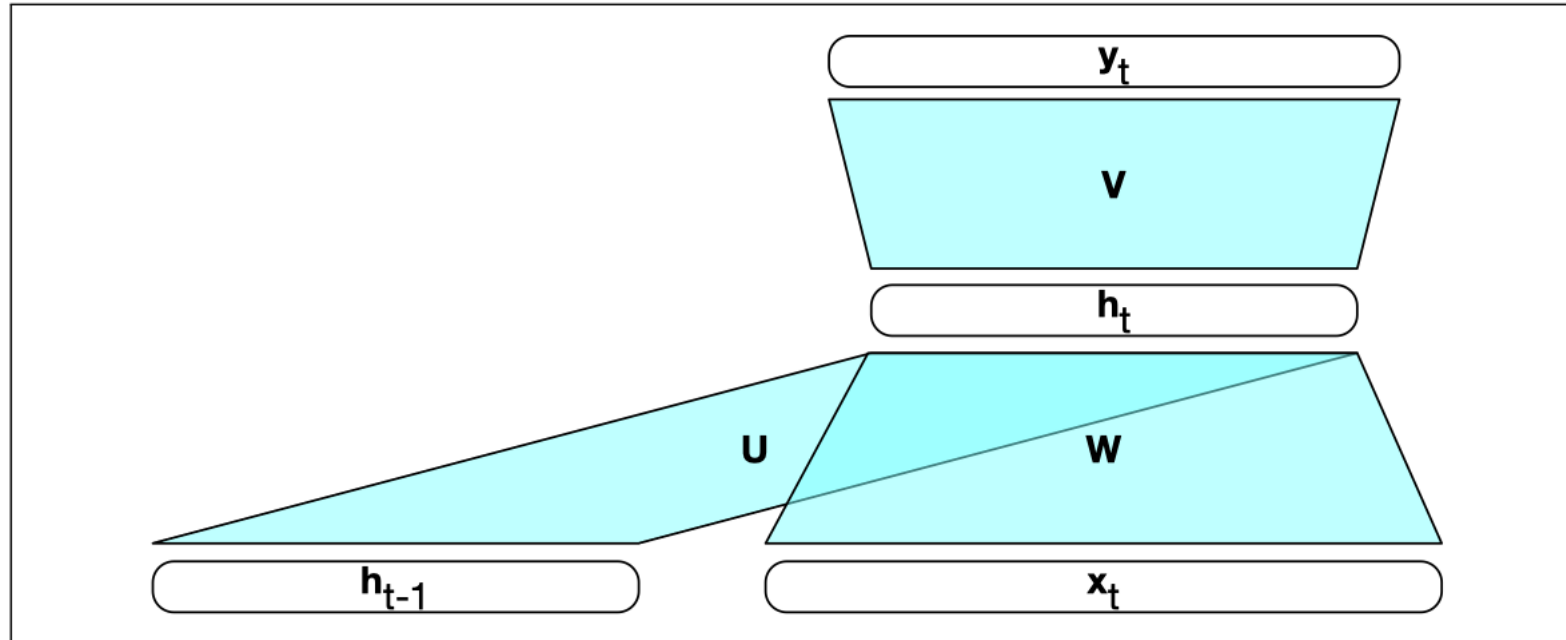
**Figure 9.1** Simple recurrent neural network after [Elman \(1990\)](#). The hidden layer includes a recurrent connection as part of its input. That is, the activation value of the hidden layer depends on the current input as well as the activation value of the hidden layer from the previous time step.



# Explaining the Figure

- As with ordinary feedforward networks, an input vector representing the current input,  $x_t$ , is multiplied by a weight matrix and then passed through a non-linear activation function to compute the values for a layer of hidden units. This hidden layer is then used to calculate a cor output,  $y_t$ .
- In a departure from our earlier window-based approach, sequences are processed by presenting one item at a time to the network. We'll use subscripts to represent time, thus  $x_t$  will mean the input vector  $x$  at time  $t$ .
- The key difference from a feedforward network lies in the recurrent link shown in the figure with the dashed line. This link augments the input to the computation at the hidden layer with the value of the hidden layer from the preceding point in time.

# RNN as a standard FFN



**Figure 9.2** Simple recurrent neural network illustrated as a feedforward network.

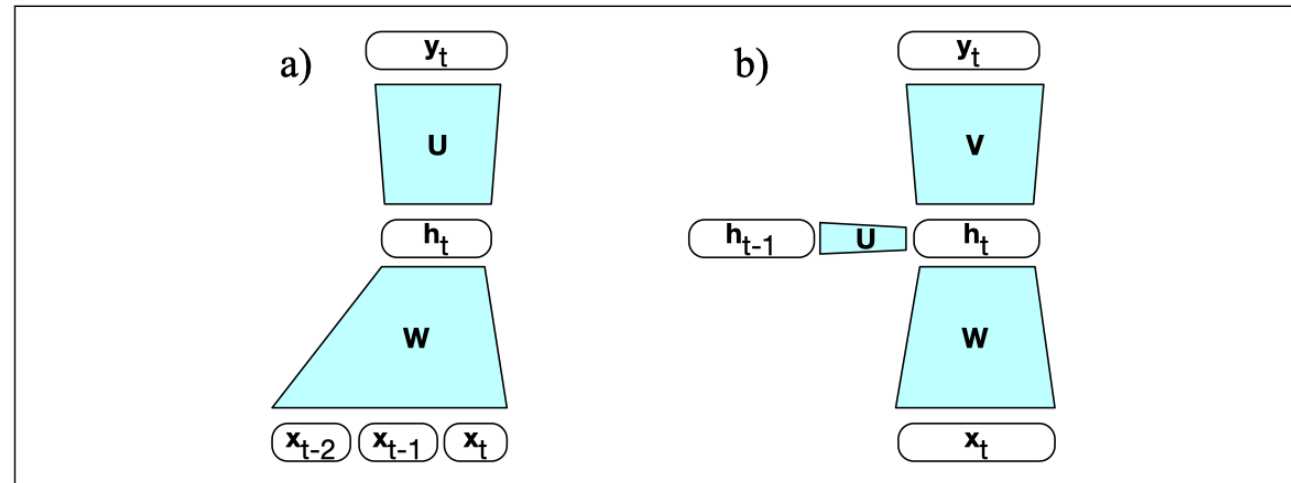
# Explaining the Figure

- To compute an output  $y_t$  for an input  $x_t$ , we need the activation value for the hidden layer  $h_t$ .
- To calculate this, we multiply the input  $x_t$  with the weight matrix  $W$ , and the hidden layer from the previous time step  $h_{t-1}$  with the weight matrix  $U$ .
- We add these values together and pass them through a suitable activation function,  $g$ , to arrive at the activation value for the current hidden layer,  $h_t$ . Once we have the values for the hidden layer, we proceed with the usual computation to generate the output vector.

$$\mathbf{h}_t = g(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t)$$

$$\mathbf{y}_t = f(\mathbf{V}\mathbf{h}_t)$$

# FFN vs RNN



**Figure 9.5** Simplified sketch of (a) a feedforward neural language model versus (b) an RNN language model moving through a text.

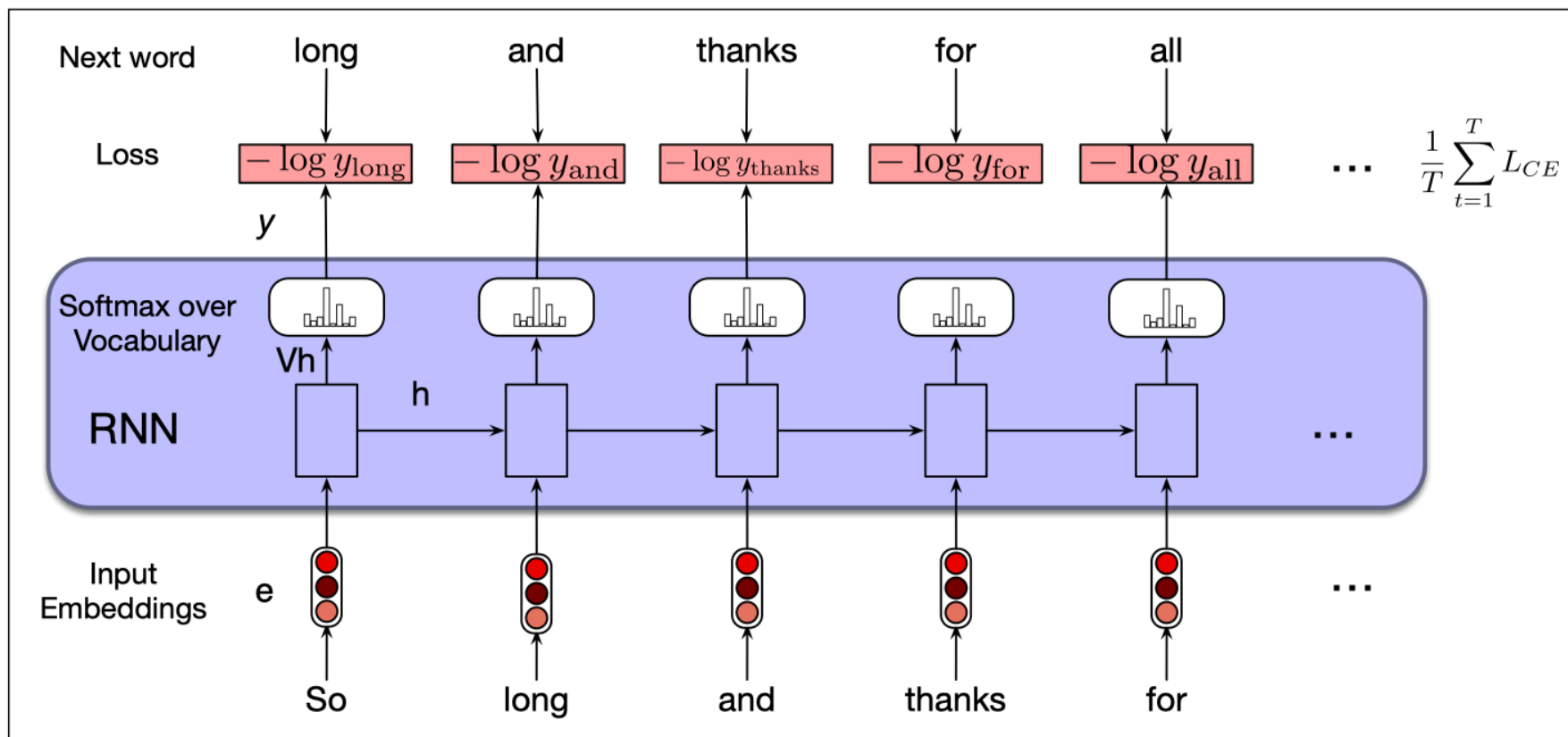
distribution over the entire vocabulary. That is, at time  $t$ :

$$\mathbf{e}_t = \mathbf{E}\mathbf{x}_t \quad (9.4)$$

$$\mathbf{h}_t = g(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{e}_t) \quad (9.5)$$

$$\mathbf{y}_t = \text{softmax}(\mathbf{V}\mathbf{h}_t) \quad (9.6)$$

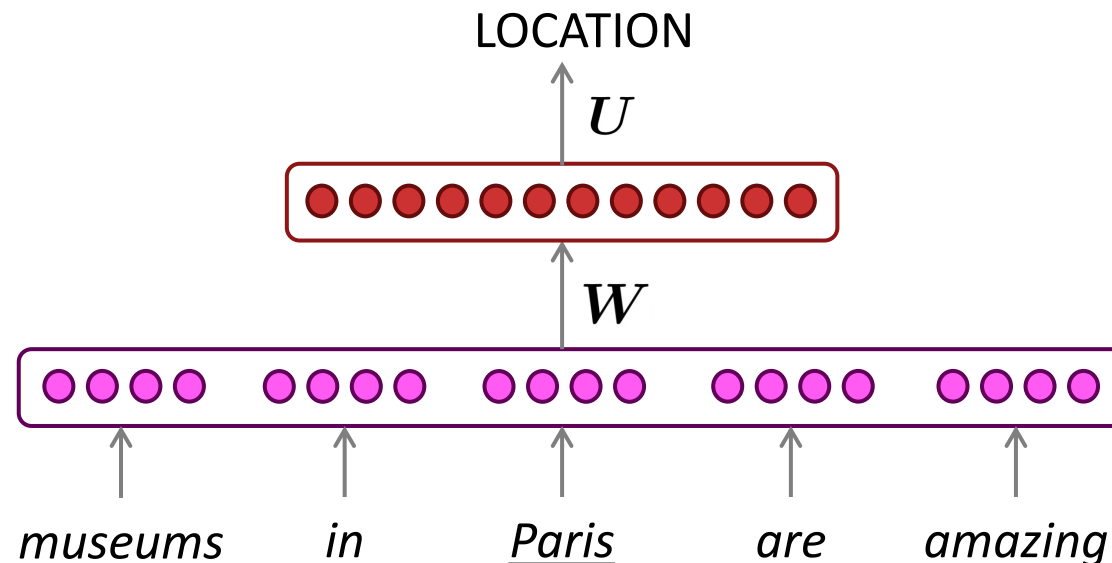
# Training RNNs



**Figure 9.6** Training RNNs as language models.

# How to build a *neural* language model?

- Recall the Language Modeling task:
  - Input: sequence of words  $x^{(1)}, x^{(2)}, \dots, x^{(t)}$
  - Output: prob. dist. of the next word  $P(x^{(t+1)} | x^{(t)}, \dots, x^{(1)})$
- How about a **window-based neural model**?
  - We saw this applied to Named Entity Recognition earlier



# A fixed-window neural Language Model

~~as the proctor started the clock~~  
discard

the students opened their \_\_\_\_\_  
fixed window

# A fixed-window neural Language Model

output distribution

$$\hat{y} = \text{softmax}(U\mathbf{h} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

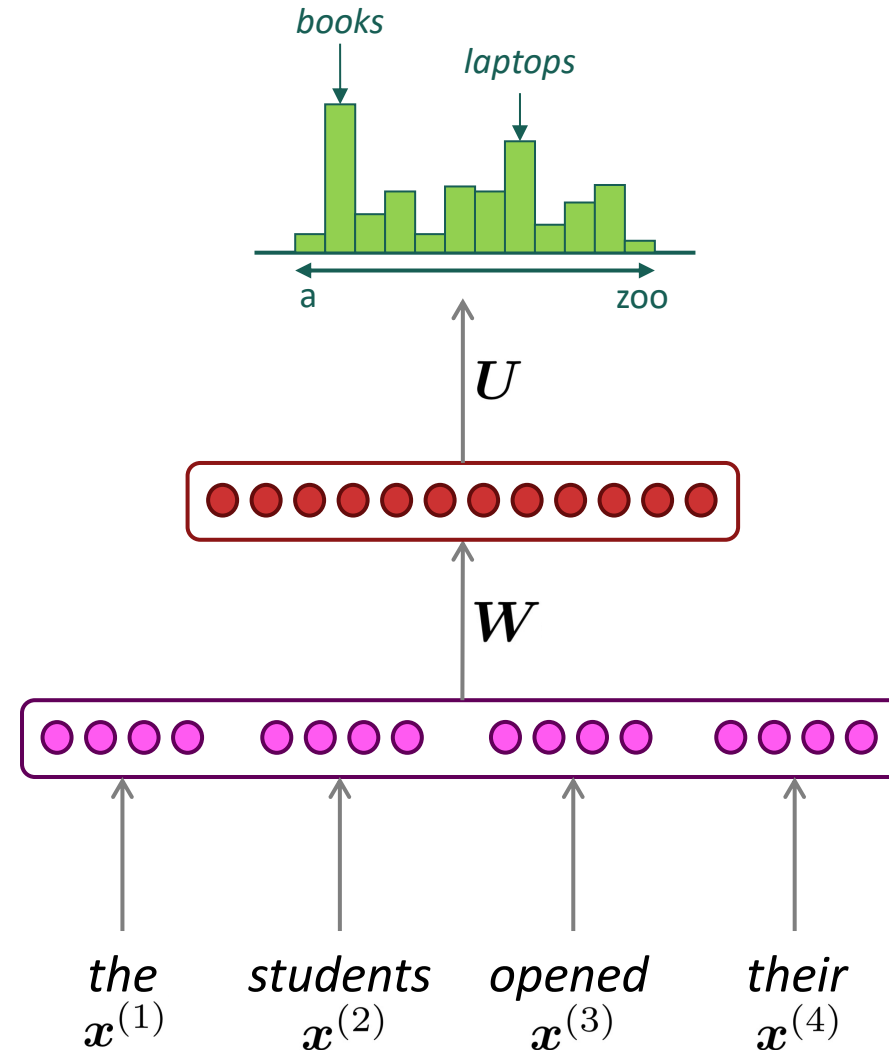
$$\mathbf{h} = f(\mathbf{W}\mathbf{e} + \mathbf{b}_1)$$

concatenated word embeddings

$$\mathbf{e} = [\mathbf{e}^{(1)}; \mathbf{e}^{(2)}; \mathbf{e}^{(3)}; \mathbf{e}^{(4)}]$$

words / one-hot vectors

$$\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}$$





# A fixed-window neural Language Model

Approximately: Y. Bengio, et al. (2000/2003): A Neural Probabilistic Language Model

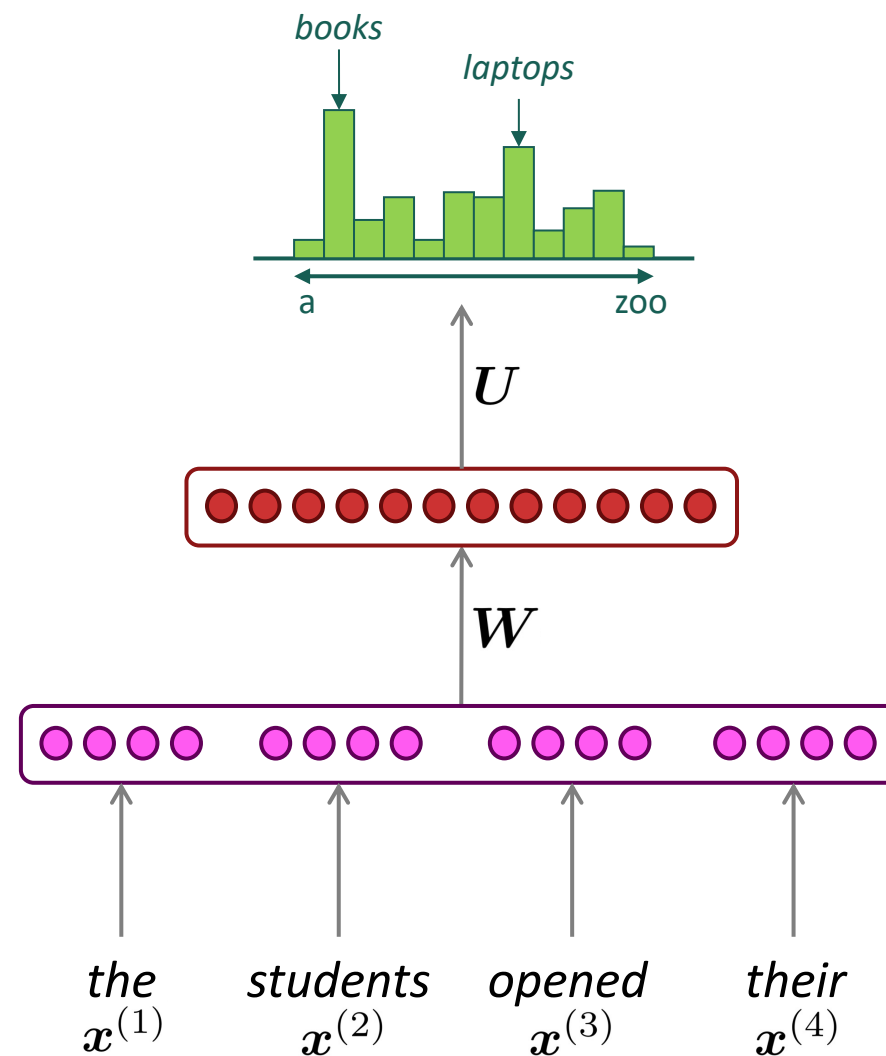
**Improvements** over  $n$ -gram LM:

- No sparsity problem
- Don't need to store all observed  $n$ -grams

Remaining **problems**:

- Fixed window is **too small**
  - Enlarging window enlarges  $W$
  - Window can never be large enough!
  - $x^{(1)}$  and  $x^{(2)}$  are multiplied by completely different weights in  $W$ .
- No symmetry** in how the inputs are processed.

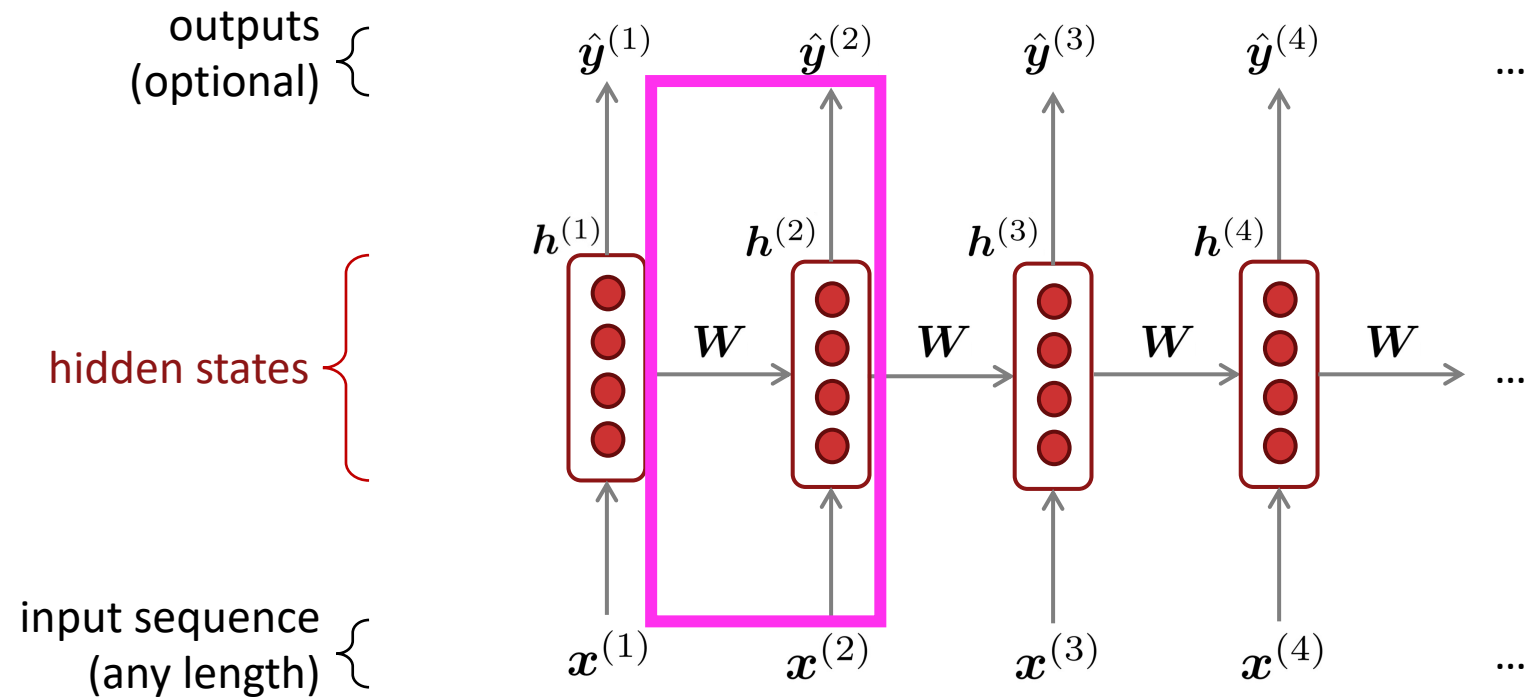
We need a neural architecture  
that can process *any length input*



# 3. Recurrent Neural Networks (RNN)

A family of neural architectures

**Core idea:** Apply the same weights  $W$  repeatedly



# A Simple RNN Language Model

output distribution

$$\hat{\mathbf{y}}^{(t)} = \text{softmax} \left( \mathbf{U} \mathbf{h}^{(t)} + \mathbf{b}_2 \right) \in \mathbb{R}^{|V|}$$

hidden states

$$\mathbf{h}^{(t)} = \sigma \left( \mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_e \mathbf{e}^{(t)} + \mathbf{b}_1 \right)$$

$\mathbf{h}^{(0)}$  is the initial hidden state

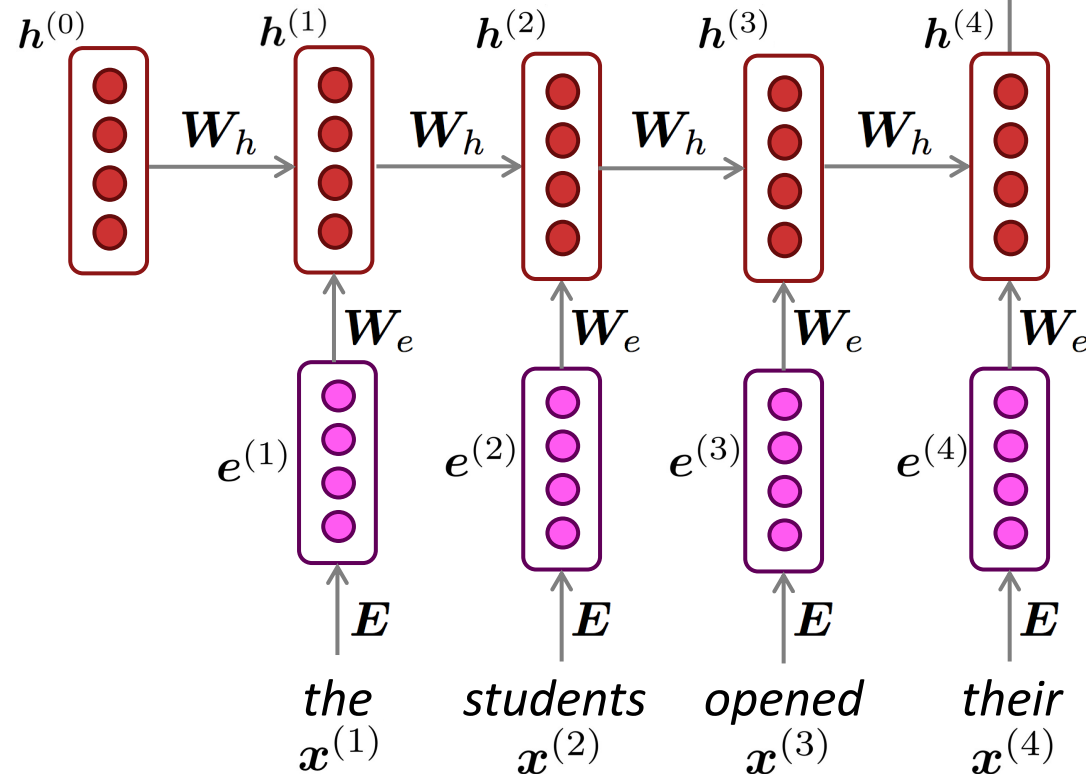
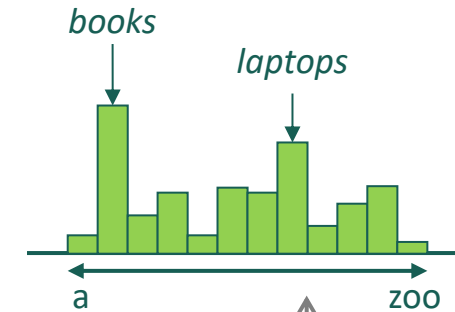
word embeddings

$$\mathbf{e}^{(t)} = \mathbf{E} \mathbf{x}^{(t)}$$

words / one-hot vectors

$$\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$$

$$\hat{\mathbf{y}}^{(4)} = P(\mathbf{x}^{(5)} | \text{the students opened their})$$



Note: this input sequence could be much longer now!

# RNN Language Models

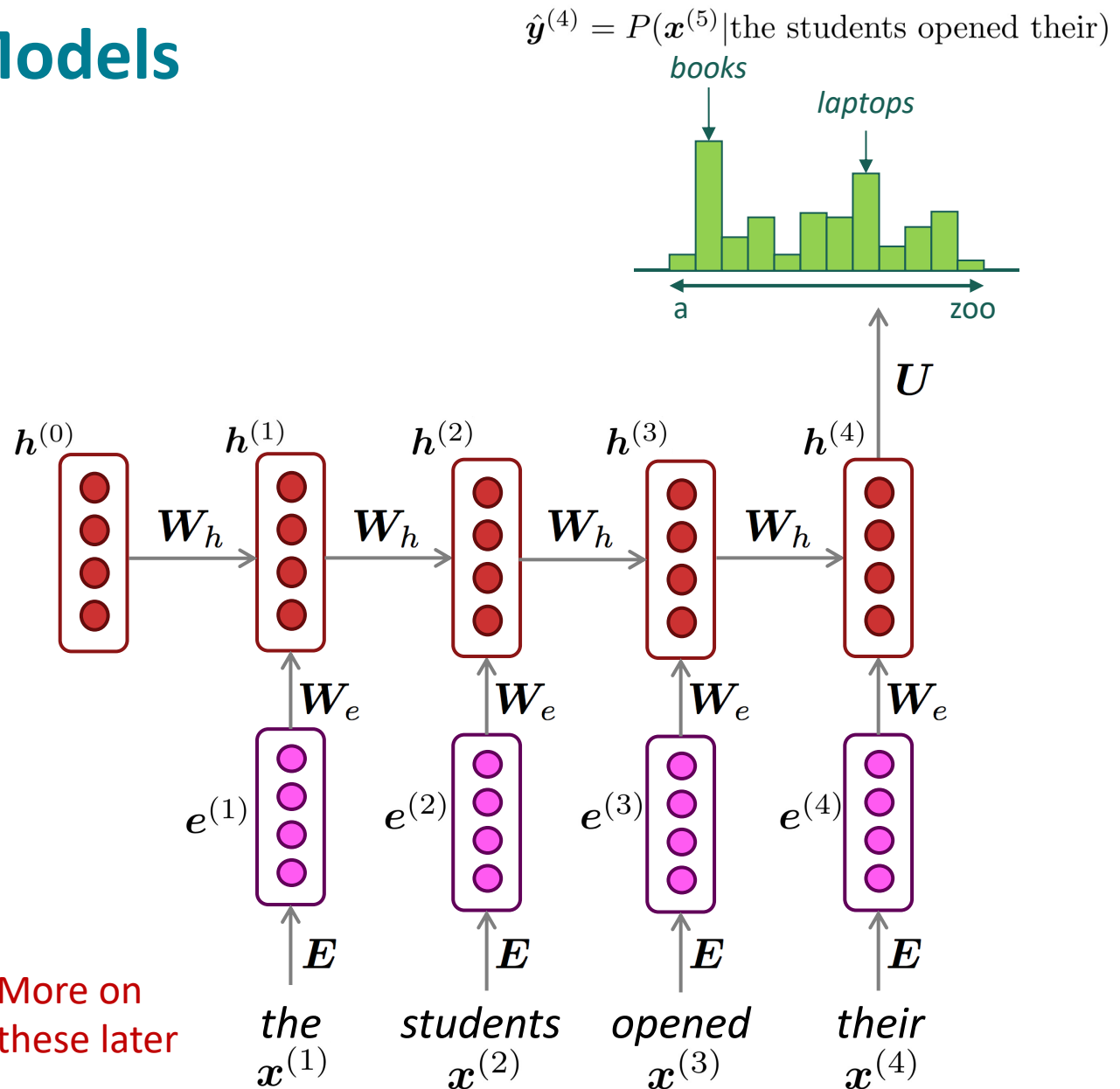
## RNN Advantages:

- Can process **any length** input
- Computation for step  $t$  can (in theory) use information from **many steps back**
- **Model size doesn't increase** for longer input context
- Same weights applied on every timestep, so there is **symmetry** in how inputs are processed.

## RNN Disadvantages:

- Recurrent computation is **slow**
- In practice, difficult to access information from **many steps back**

More on these later



# Training an RNN Language Model

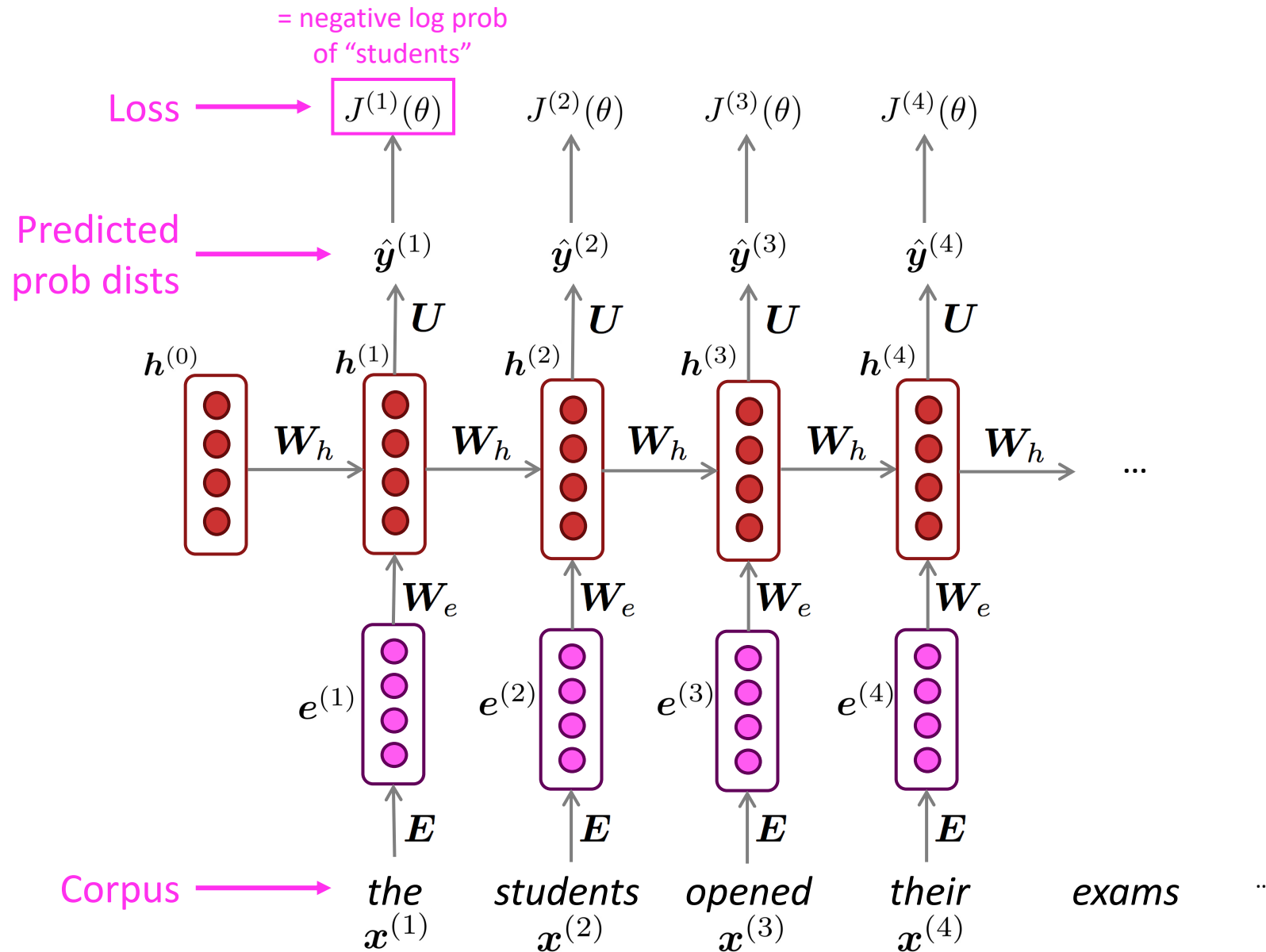
- Get a **big corpus of text** which is a sequence of words  $x^{(1)}, \dots, x^{(T)}$
- Feed into RNN-LM; compute output distribution  $\hat{\mathbf{y}}^{(t)}$  **for every step  $t$** .
  - i.e., predict probability dist of *every word*, given words so far
- **Loss function** on step  $t$  is **cross-entropy** between predicted probability distribution  $\hat{\mathbf{y}}^{(t)}$ , and the true next word  $\mathbf{y}^{(t)}$  (one-hot for  $x^{(t+1)}$ ):

$$J^{(t)}(\theta) = CE(\mathbf{y}^{(t)}, \hat{\mathbf{y}}^{(t)}) = - \sum_{w \in V} \mathbf{y}_w^{(t)} \log \hat{\mathbf{y}}_w^{(t)} = - \log \hat{\mathbf{y}}_{x_{t+1}}^{(t)}$$

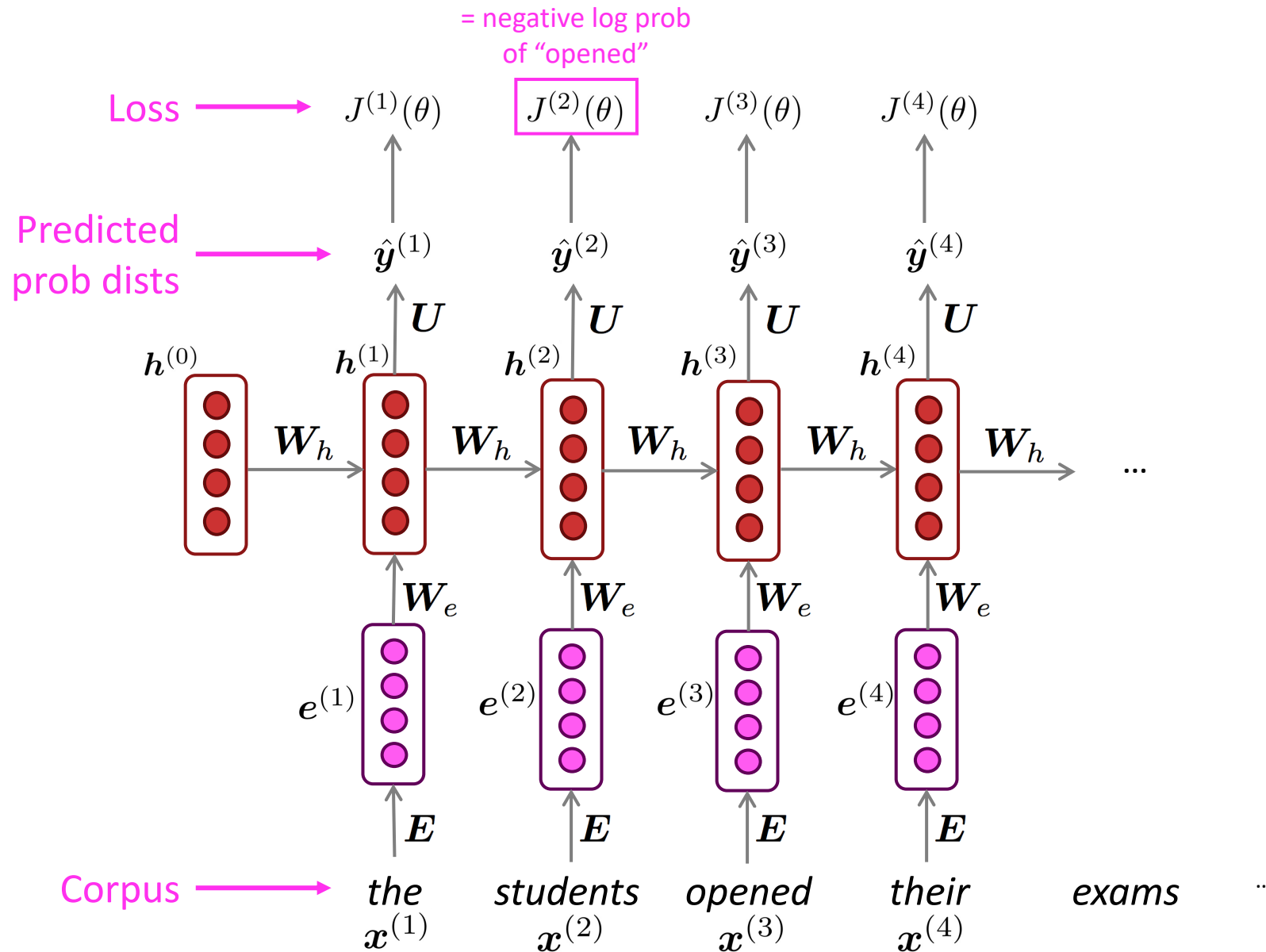
- Average this to get **overall loss** for entire training set:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^T J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^T - \log \hat{\mathbf{y}}_{x_{t+1}}^{(t)}$$

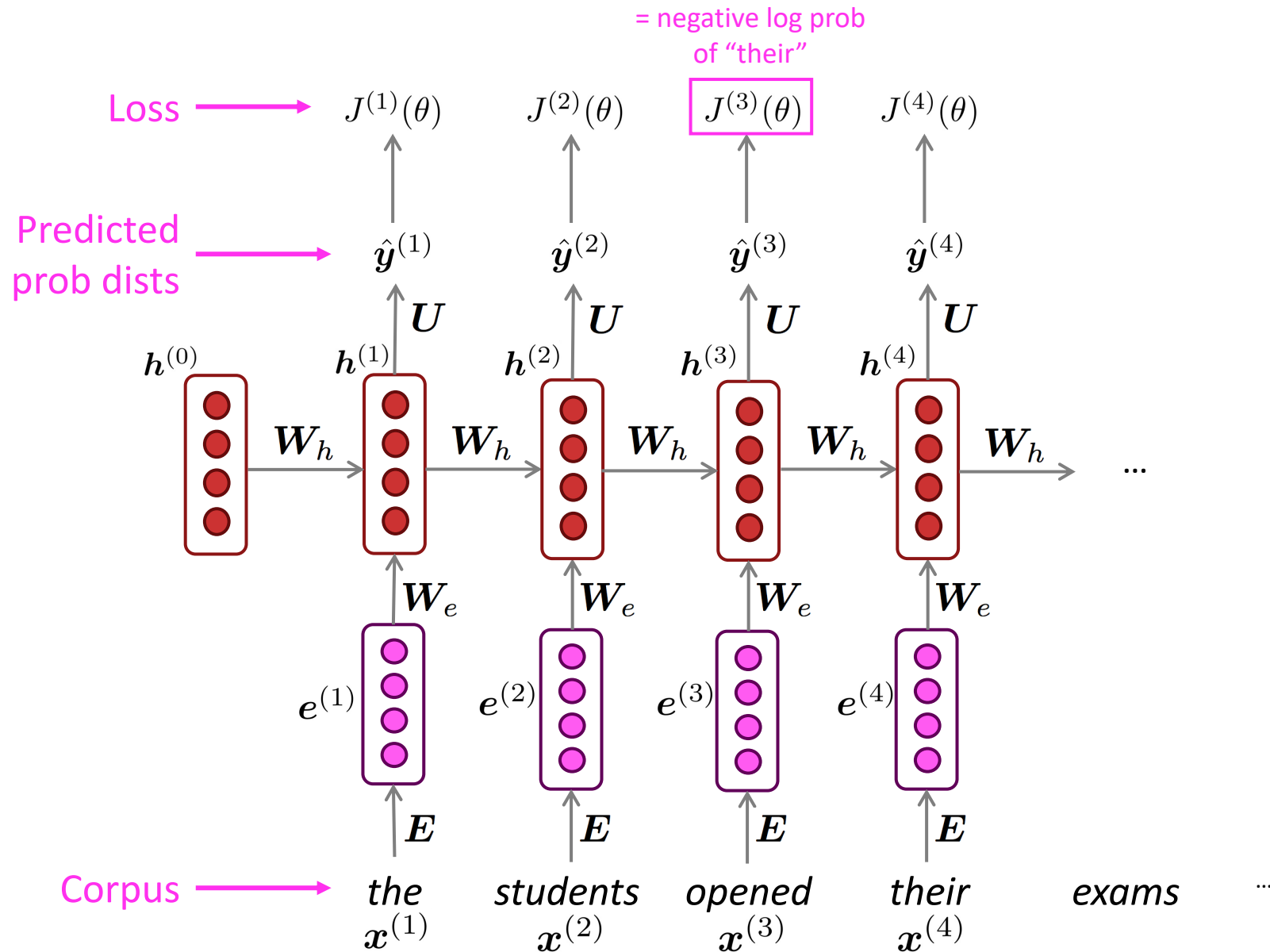
# Training an RNN Language Model



# Training an RNN Language Model

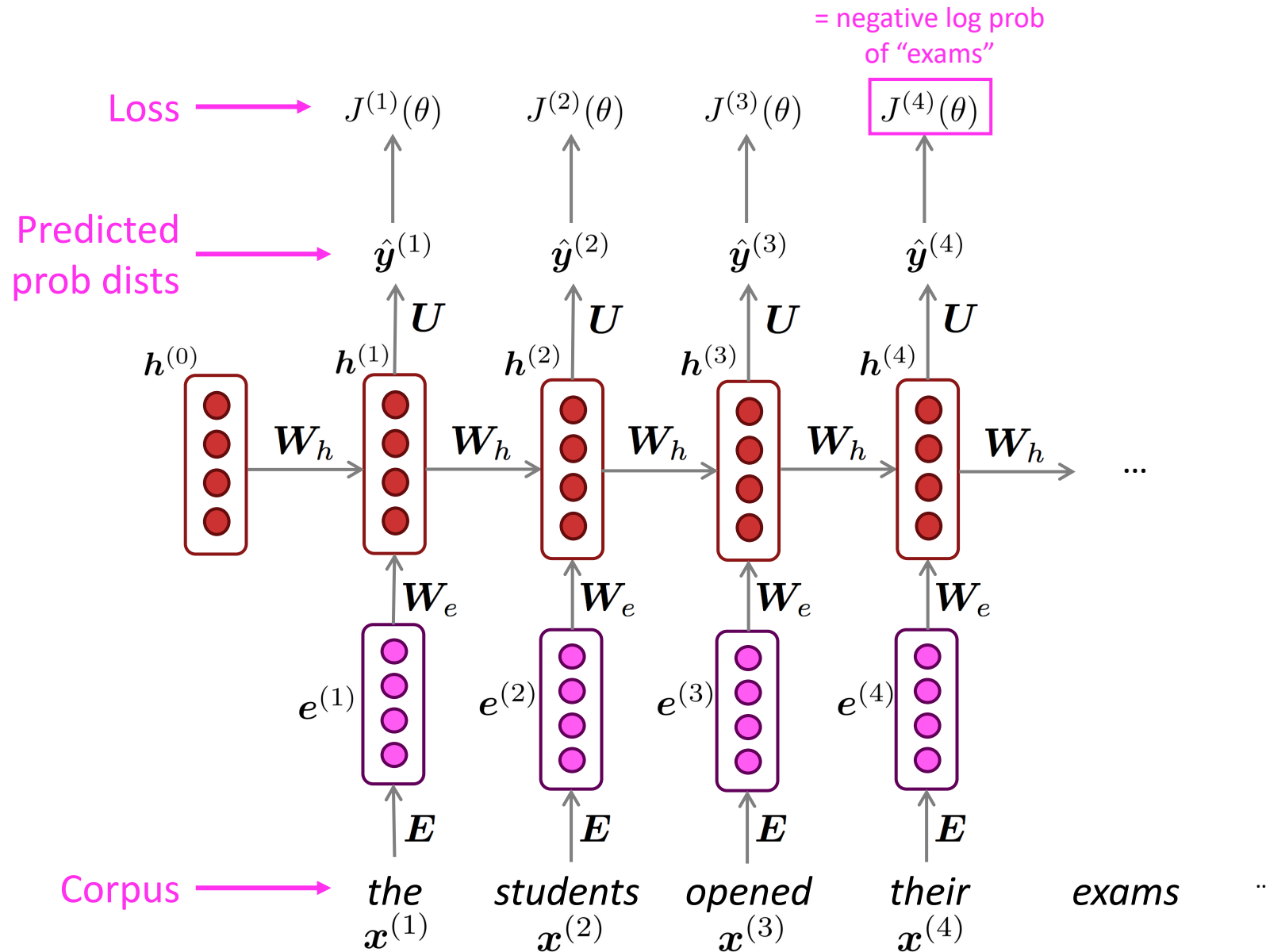


# Training an RNN Language Model



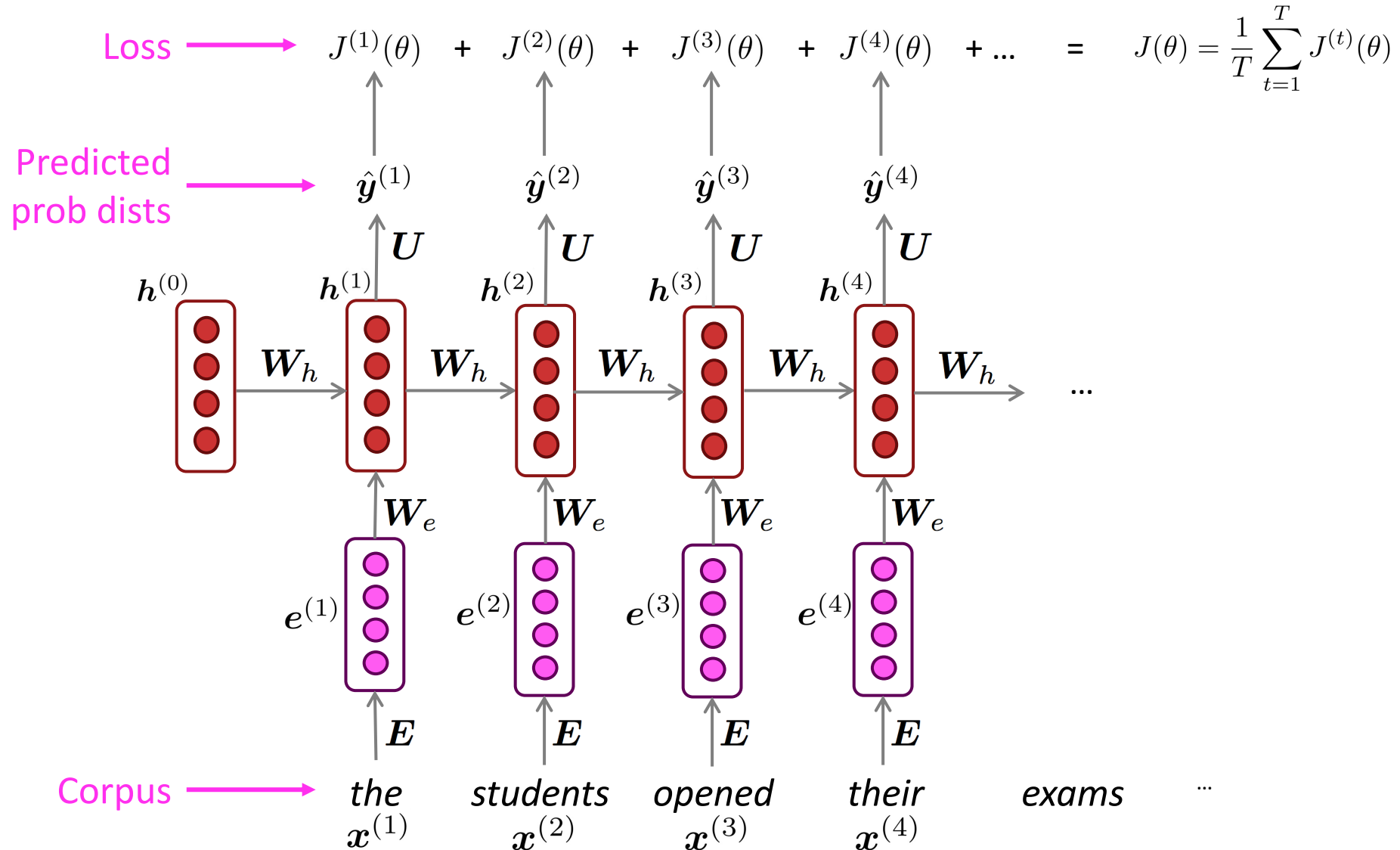


# Training an RNN Language Model



# Training an RNN Language Model

“Teacher forcing”



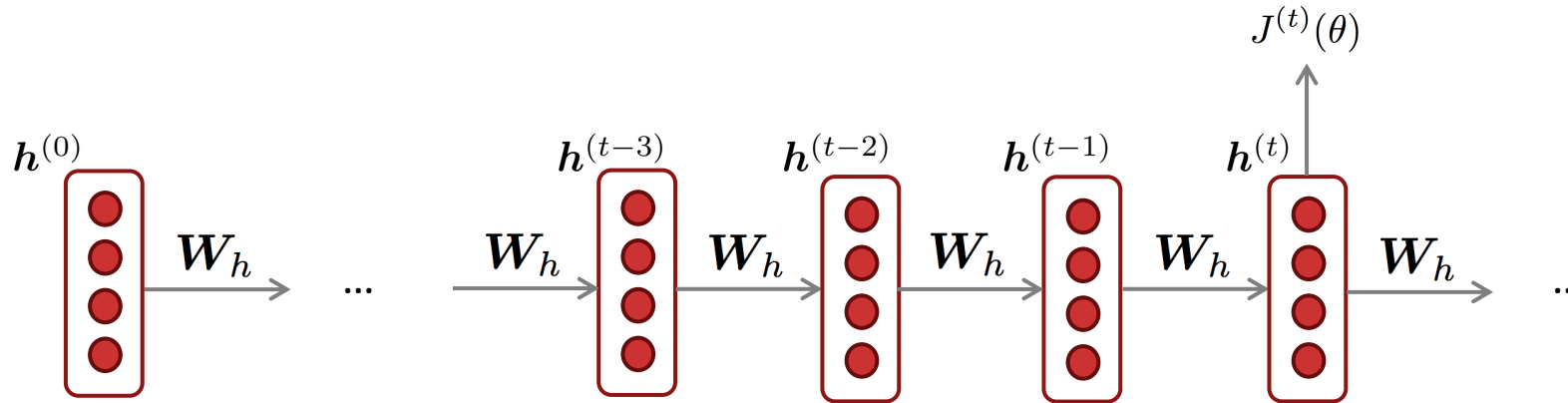
# Training a RNN Language Model

- However: Computing loss and gradients across **entire corpus**  $x^{(1)}, \dots, x^{(T)}$  at once is **too expensive** (memory-wise)!

$$J(\theta) = \frac{1}{T} \sum_{t=1}^T J^{(t)}(\theta)$$

- In practice, consider  $x^{(1)}, \dots, x^{(T)}$  as a **sentence** (or a **document**)
- Recall: **Stochastic Gradient Descent** allows us to compute loss and gradients for small chunk of data, and update.
- Compute loss  $J(\theta)$  for a sentence (actually, a batch of sentences), compute gradients and update weights. Repeat on a new batch of sentences.

# Backpropagation for RNNs



**Question:** What's the derivative of  $J^{(t)}(\theta)$  w.r.t. the **repeated** weight matrix  $W_h$  ?

**Answer:** 
$$\frac{\partial J^{(t)}}{\partial W_h} = \sum_{i=1}^t \frac{\partial J^{(t)}}{\partial W_h} \Big|_{(i)}$$

“The gradient w.r.t. a repeated weight is the sum of the gradient w.r.t. each time it appears”

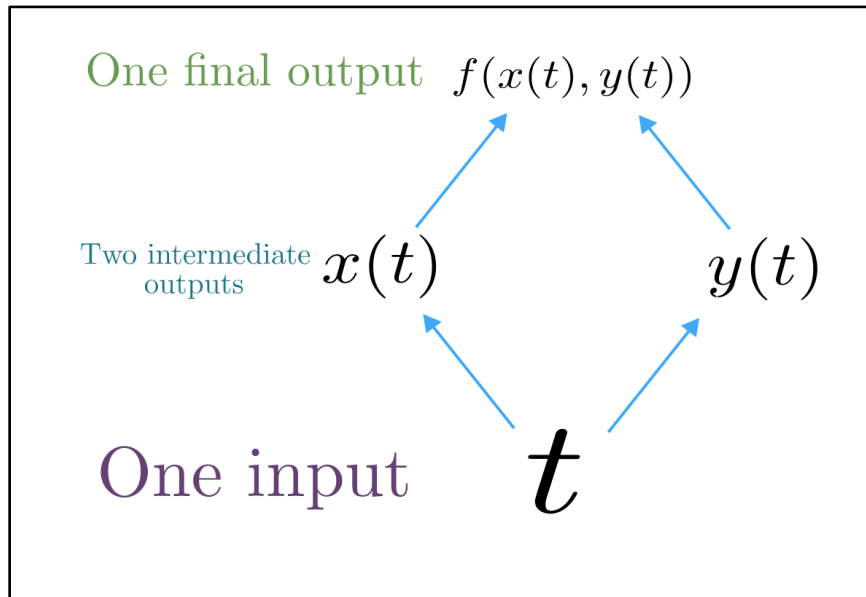
Why?

# Multivariable Chain Rule

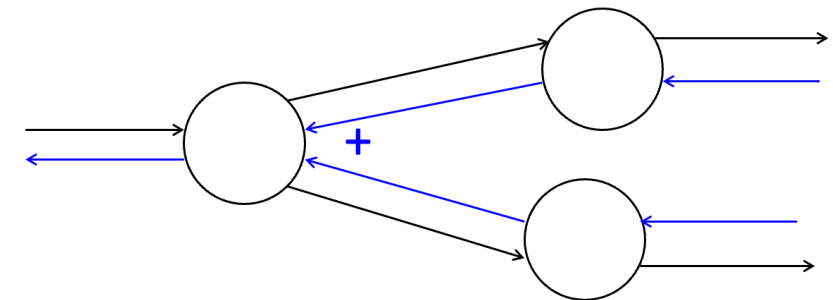
- Given a multivariable function  $f(x, y)$ , and two single variable functions  $x(t)$  and  $y(t)$ , here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(x(t), y(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

Derivative of composition function



Gradients sum at outward branches



$$a = x + y$$

$$b = \max(y, z)$$

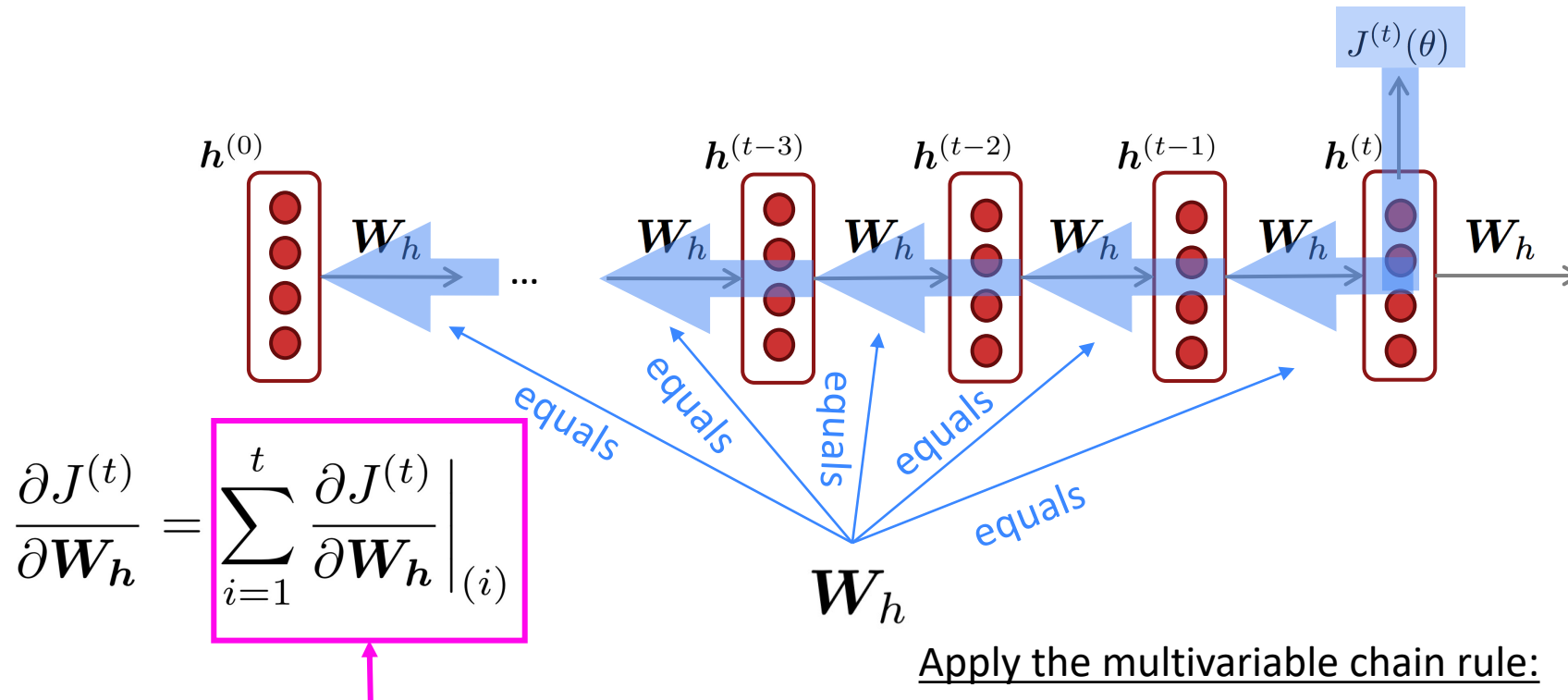
$$f = ab$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial a} \frac{\partial a}{\partial y} + \frac{\partial f}{\partial b} \frac{\partial b}{\partial y}$$

Source:

<https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version>

# Training the parameters of RNNs: Backpropagation for RNNs



In practice, often “truncated” after ~20 timesteps for training efficiency reasons

Apply the multivariable chain rule:

= 1

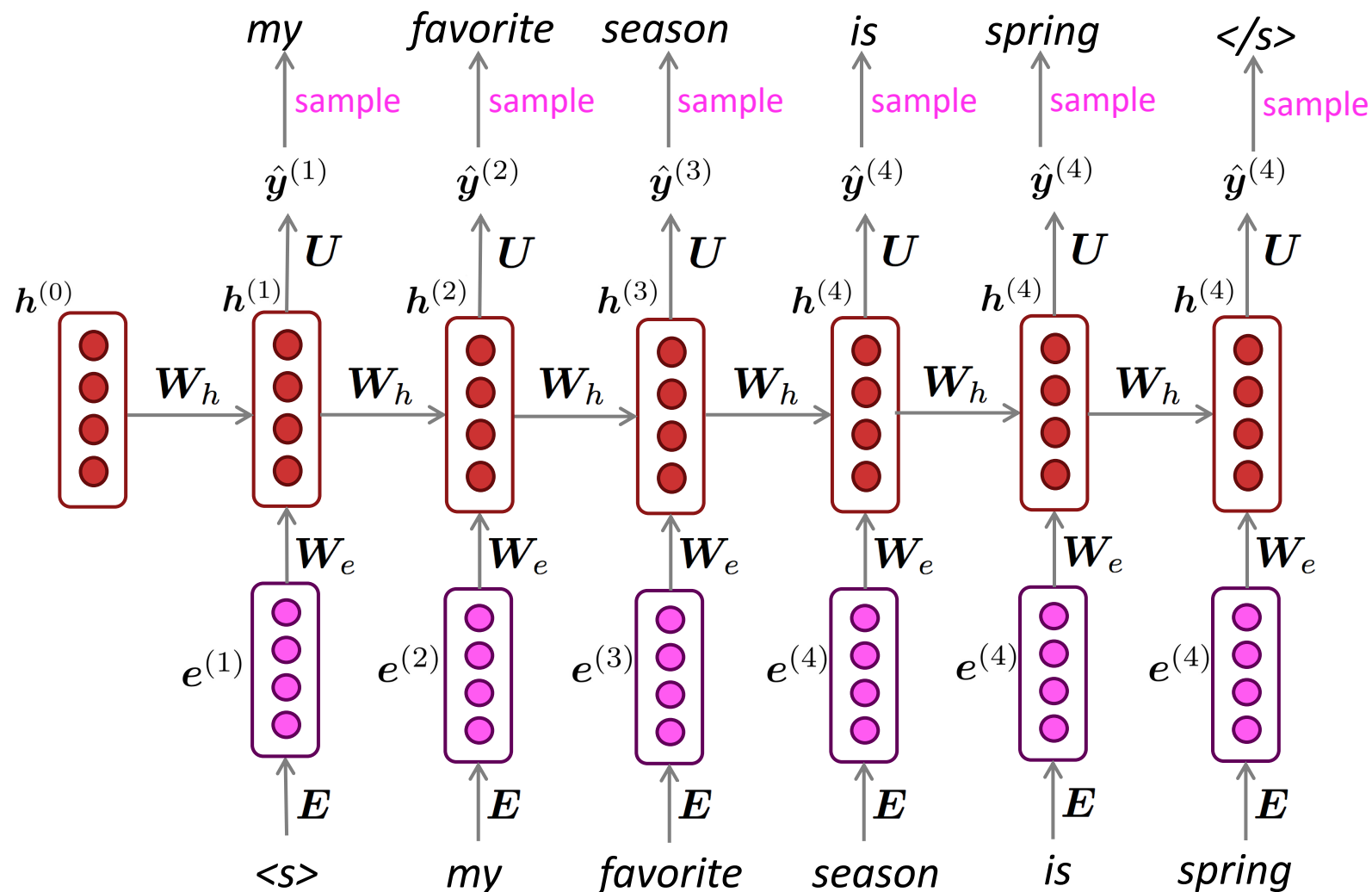
$$\begin{aligned} \frac{\partial J^{(t)}}{\partial W_h} &= \sum_{i=1}^t \frac{\partial J^{(t)}}{\partial W_h} \Big|_{(i)} \frac{\partial W_h \Big|_{(i)}}{\partial W_h} \\ &= \sum_{i=1}^t \frac{\partial J^{(t)}}{\partial W_h} \Big|_{(i)} \end{aligned}$$

**Question:** How do we calculate this?

**Answer:** Backpropagate over timesteps  $i = t, \dots, 0$ , summing gradients as you go. This algorithm is called “**backpropagation through time**” [Werbos, P.G., 1988, *Neural Networks 1*, and others]

# Generating with an RNN Language Model (“Generating roll outs”)

Just like an n-gram Language Model, you can use a RNN Language Model to **generate text** by **repeated sampling**. Sampled output becomes next step's input.



# Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on **Obama speeches**:



*The United States will step up to the cost of a new challenges of the American people that will share the fact that we created the problem. They were attacked and so that they have to say that all the task of the final days of war that I will not be able to get this done.*

**Source:** <https://medium.com/@samim/obama-rnn-machine-generated-political-speeches-c8abd18a2ea0>



# Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on *Harry Potter*:



“Sorry,” Harry shouted, panicking—“I’ll leave those brooms in London, are they?”

“No idea,” said Nearly Headless Nick, casting low close by Cedric, carrying the last bit of treacle Charms, from Harry’s shoulder, and to answer him the common room perched upon it, four arms held a shining knob from when the spider hadn’t felt it seemed. He reached the teams too.

**Source:** <https://medium.com/deep-writing/harry-potter-written-by-artificial-intelligence-8a9431803da6>

# Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on **recipes**:

Title: CHOCOLATE RANCH BARBECUE  
Categories: Game, Casseroles, Cookies, Cookies  
Yield: 6 Servings

2 tb Parmesan cheese -- chopped  
1 c Coconut milk  
3 Eggs, beaten

Place each pasta over layers of lumps. Shape mixture into the moderate oven and simmer until firm. Serve hot in bodied fresh, mustard, orange and cheese.

Combine the cheese and salt together the dough in a large skillet; add the ingredients and stir in the chocolate and pepper.









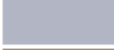















**Source:** <https://gist.github.com/nylki/1efbaa36635956d35bcc>

# Generating text with a RNN Language Model

Let's have some fun!

- You can train a RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on **paint color names**:

	Ghasty Pink 231 137 165		Sand Dan 201 172 143
	Power Gray 151 124 112		Grade Bat 48 94 83
	Navel Tan 199 173 140		Light Of Blast 175 150 147
	Bock Coe White 221 215 236		Grass Bat 176 99 108
	Horble Gray 178 181 196		Sindis Poop 204 205 194
	Homestar Brown 133 104 85		Dope 219 209 179
	Snader Brown 144 106 74		Testing 156 101 106
	Golder Craam 237 217 177		Stoner Blue 152 165 159
	Hurky White 232 223 215		Burple Simp 226 181 132
	Burf Pink 223 173 179		Stanky Bean 197 162 171
	Rose Hork 230 215 198		Turdly 190 164 116

This is an example of a **character-level RNN-LM** (predicts what **character** comes next)

# Evaluating Language Models

- The standard **evaluation metric** for Language Models is **perplexity**.

$$\text{perplexity} = \prod_{t=1}^T \left( \frac{1}{P_{\text{LM}}(\mathbf{x}^{(t+1)} | \mathbf{x}^{(t)}, \dots, \mathbf{x}^{(1)})} \right)^{1/T}$$

Normalized by number of words

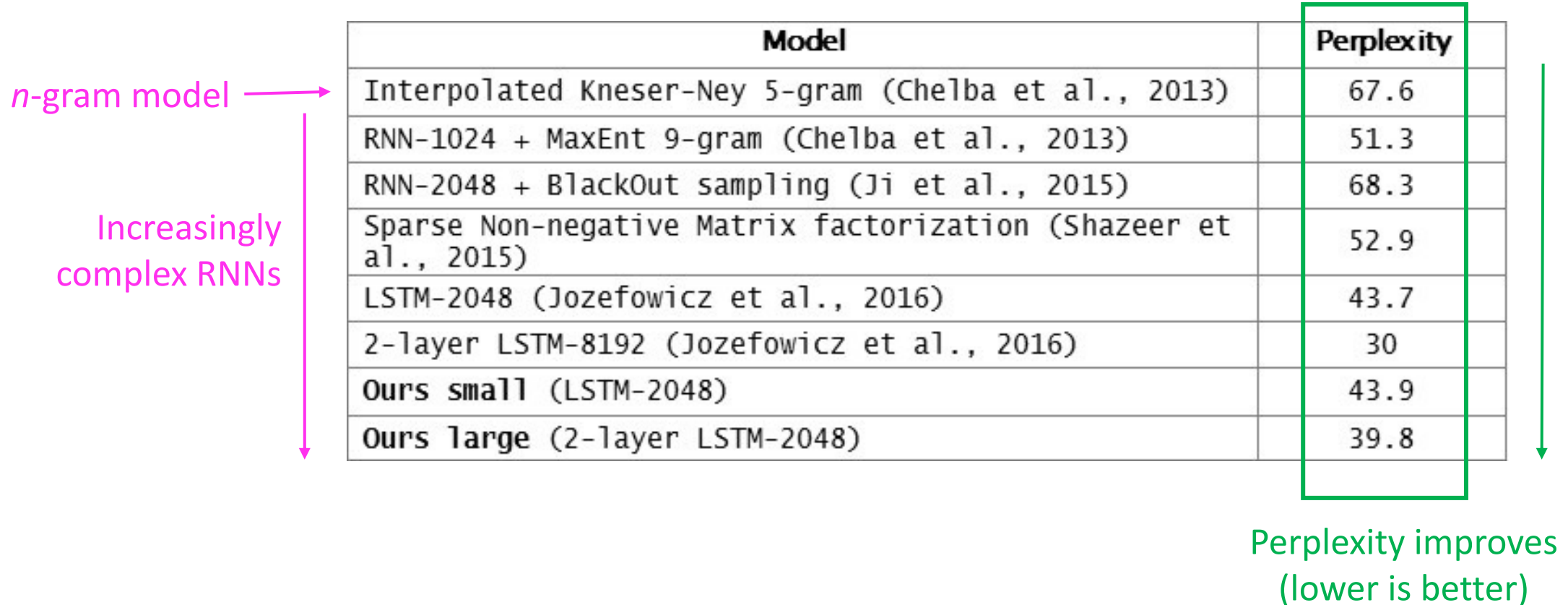
Inverse probability of corpus, according to Language Model

- This is equal to the exponential of the cross-entropy loss  $J(\theta)$  :

$$= \prod_{t=1}^T \left( \frac{1}{\hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}} \right)^{1/T} = \exp \left( \frac{1}{T} \sum_{t=1}^T -\log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)} \right) = \exp(J(\theta))$$

**Lower perplexity is better!**

# RNNs greatly improved perplexity over what came before



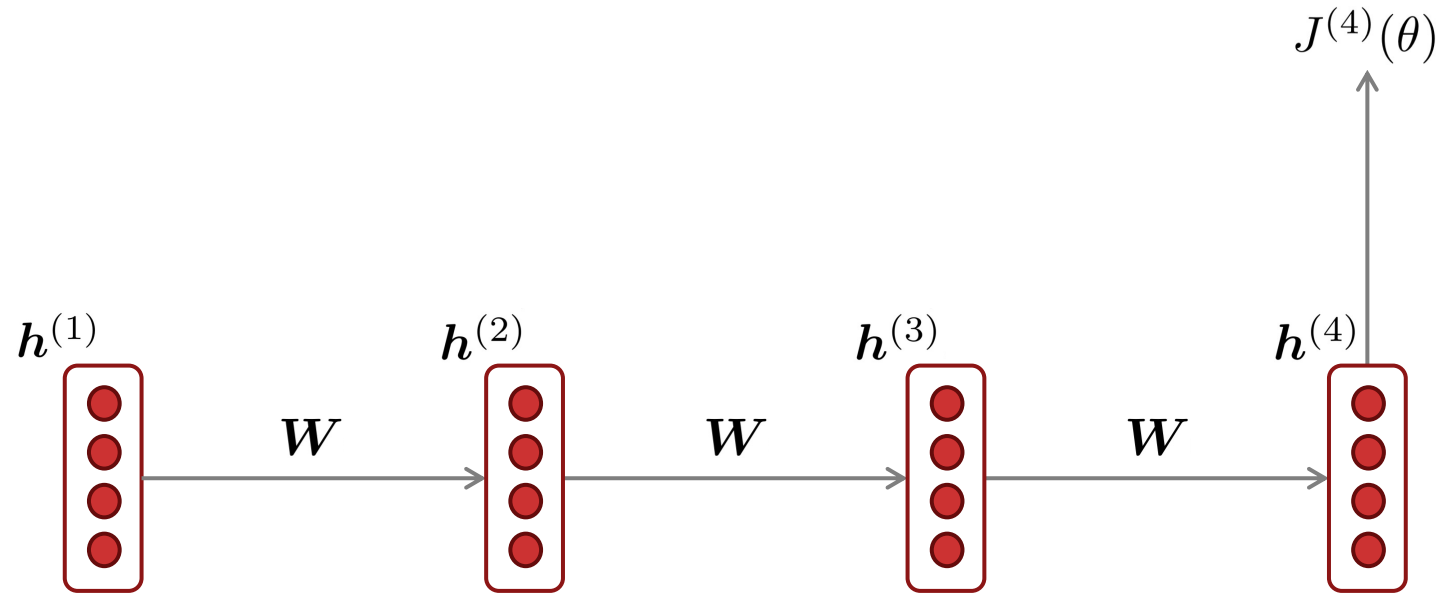
The diagram illustrates the evolution of language models. On the left, a pink arrow points downwards from the text 'n-gram model' to 'Increasingly complex RNNs', indicating a progression of model complexity. In the center is a table with two columns: 'Model' and 'Perplexity'. The table lists eight models, with the last two ('Ours small' and 'Ours large') highlighted in green. To the right of the table, a green arrow points downwards from the top row to the bottom row, indicating that as the model complexity increases, the perplexity improves (decreases).

Model	Perplexity
Interpolated Kneser-Ney 5-gram (Chelba et al., 2013)	67.6
RNN-1024 + MaxEnt 9-gram (Chelba et al., 2013)	51.3
RNN-2048 + BlackOut sampling (Ji et al., 2015)	68.3
Sparse Non-negative Matrix factorization (Shazeer et al., 2015)	52.9
LSTM-2048 (Jozefowicz et al., 2016)	43.7
2-layer LSTM-8192 (Jozefowicz et al., 2016)	30
Ours small (LSTM-2048)	43.9
Ours large (2-layer LSTM-2048)	39.8

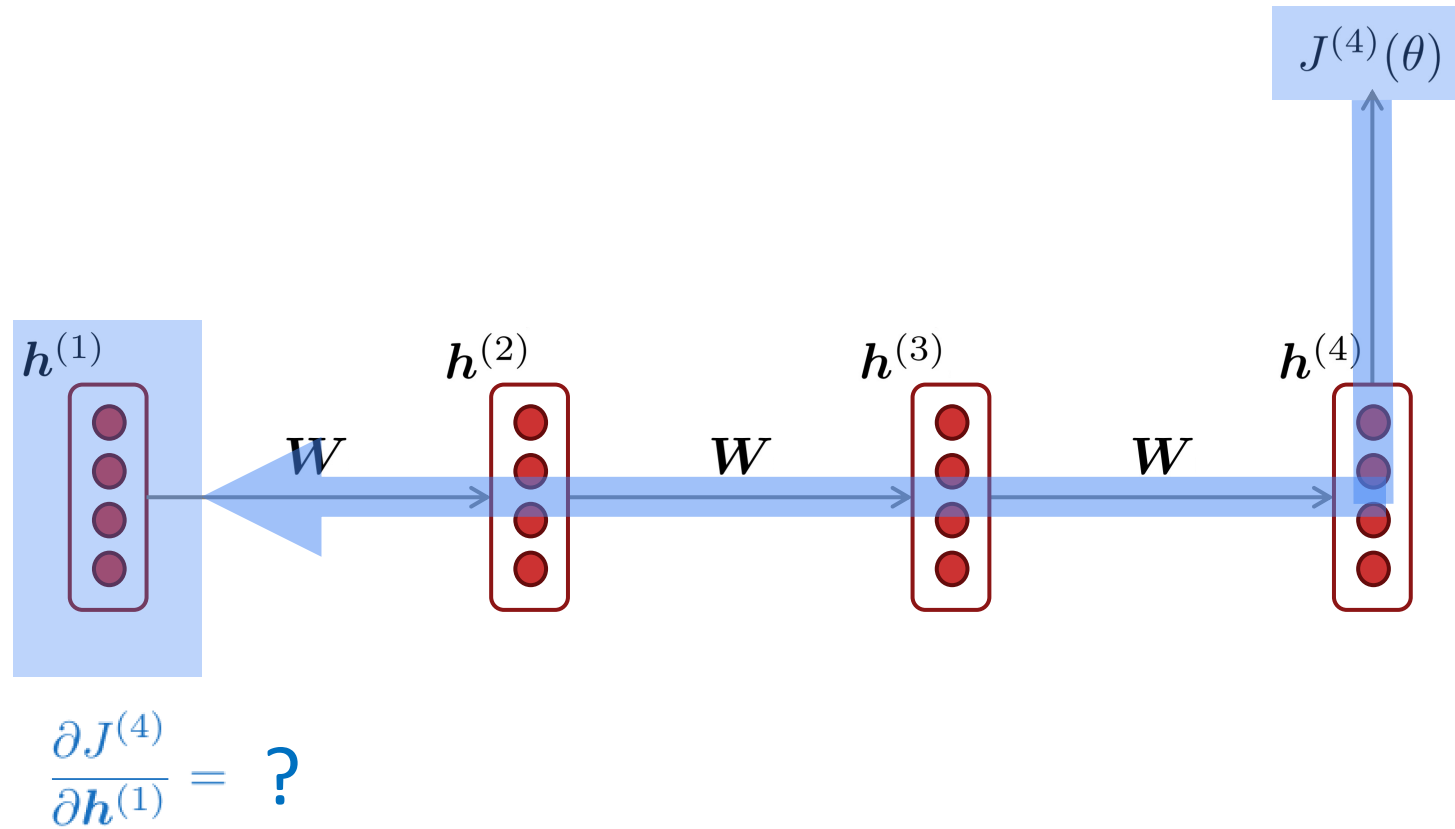
Perplexity improves (lower is better)

Source: <https://research.fb.com/building-an-efficient-neural-language-model-over-a-billion-words/>

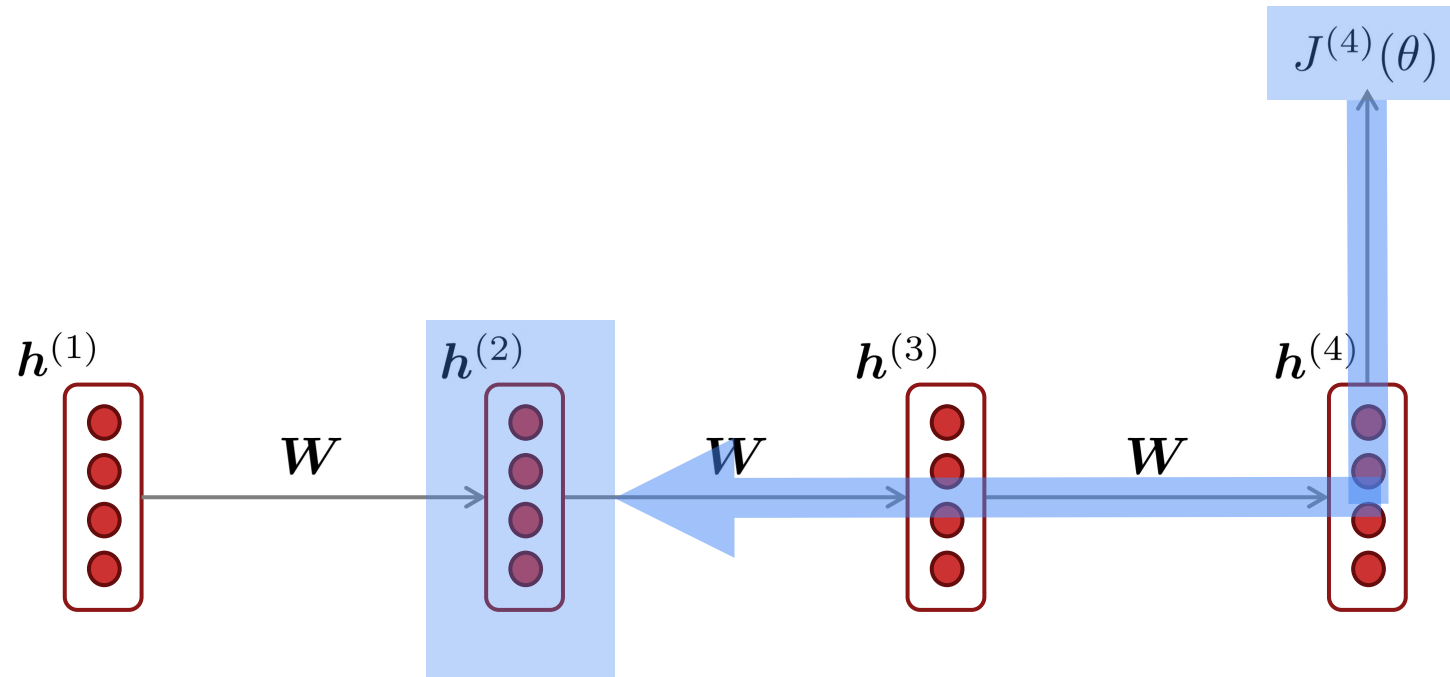
## 4. Problems with RNNs: Vanishing and Exploding Gradients



# Vanishing gradient intuition



# Vanishing gradient intuition

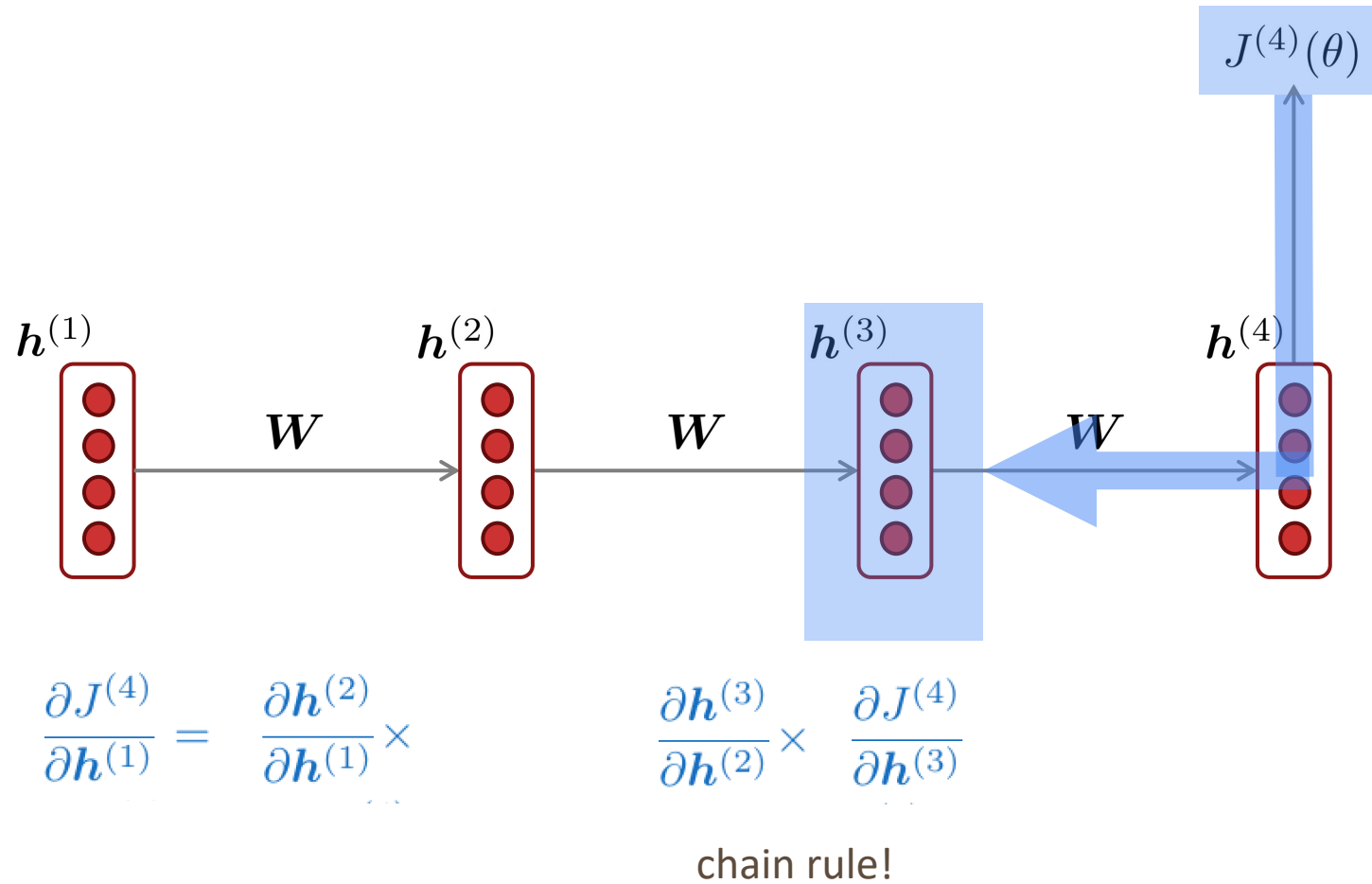


$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = \frac{\partial h^{(2)}}{\partial h^{(1)}} \times \frac{\partial J^{(4)}}{\partial h^{(2)}}$$

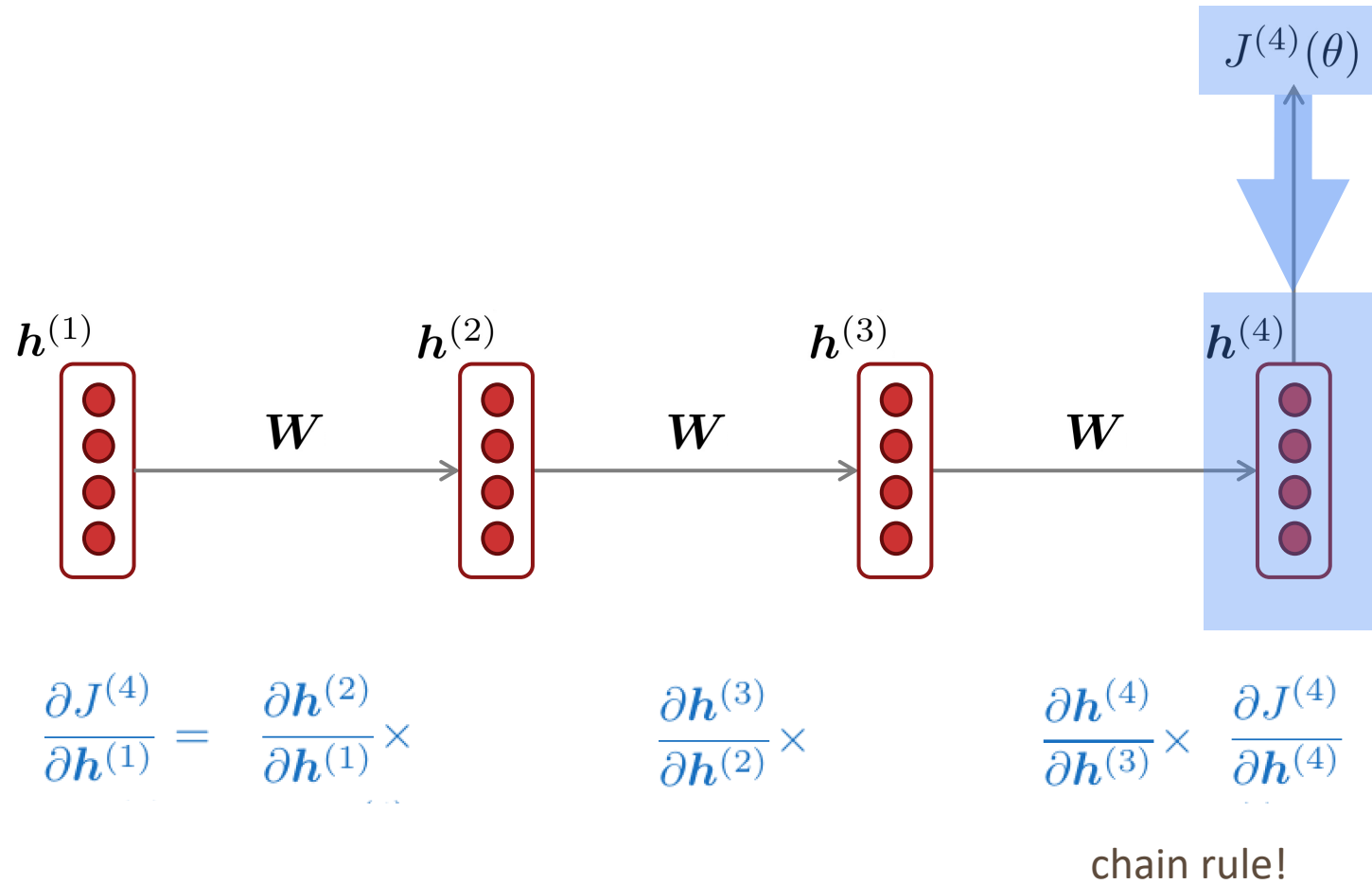
chain rule!



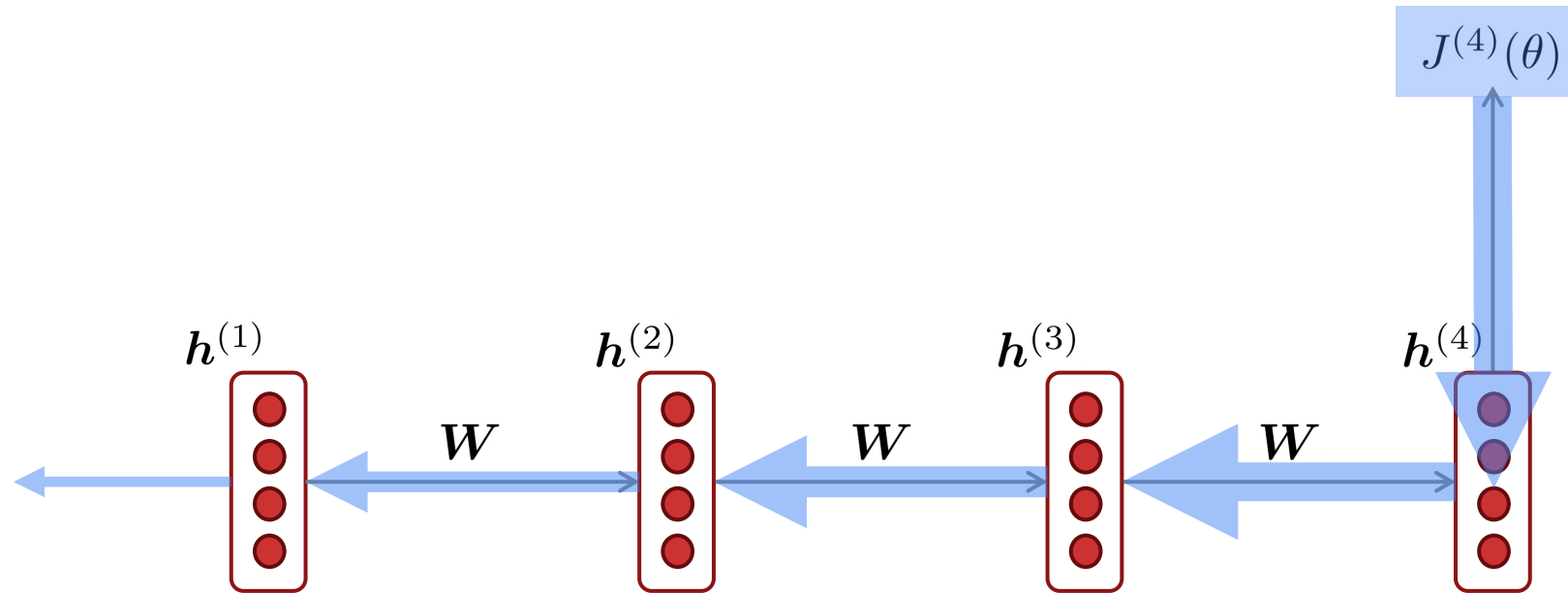
# Vanishing gradient intuition



# Vanishing gradient intuition



# Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = \boxed{\frac{\partial h^{(2)}}{\partial h^{(1)}}} \times \boxed{\frac{\partial h^{(3)}}{\partial h^{(2)}}} \times \boxed{\frac{\partial h^{(4)}}{\partial h^{(3)}}} \times \frac{\partial J^{(4)}}{\partial h^{(4)}}$$

What happens if these are small?


**Vanishing gradient problem:**  
When these are small, the gradient signal gets smaller and smaller as it backpropagates further

# Vanishing gradient proof sketch (linear case)

- Recall:
$$\mathbf{h}^{(t)} = \sigma \left( \mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_x \mathbf{x}^{(t)} + \mathbf{b}_1 \right)$$
- What if  $\sigma$  were the identity function,  $\sigma(x) = x$  ?

$$\begin{aligned} \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}} &= \text{diag} \left( \sigma' \left( \mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_x \mathbf{x}^{(t)} + \mathbf{b}_1 \right) \right) \mathbf{W}_h && \text{(chain rule)} \\ &= \mathbf{I} \mathbf{W}_h = \mathbf{W}_h \end{aligned}$$

- Consider the gradient of the loss  $J^{(i)}(\theta)$  on step  $i$ , with respect to the hidden state  $\mathbf{h}^{(j)}$  on some previous step  $j$ . Let  $\ell = i - j$

$$\begin{aligned} \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(j)}} &= \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(i)}} \prod_{j < t \leq i} \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}} && \text{(chain rule)} \\ &= \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(i)}} \prod_{j < t \leq i} \mathbf{W}_h = \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(i)}} \boxed{\mathbf{W}_h^\ell} && \text{(value of } \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}} \text{)} \end{aligned}$$


If  $\mathbf{W}_h$  is “small”, then this term gets exponentially problematic as  $\ell$  becomes large

# Vanishing gradient proof sketch (linear case)

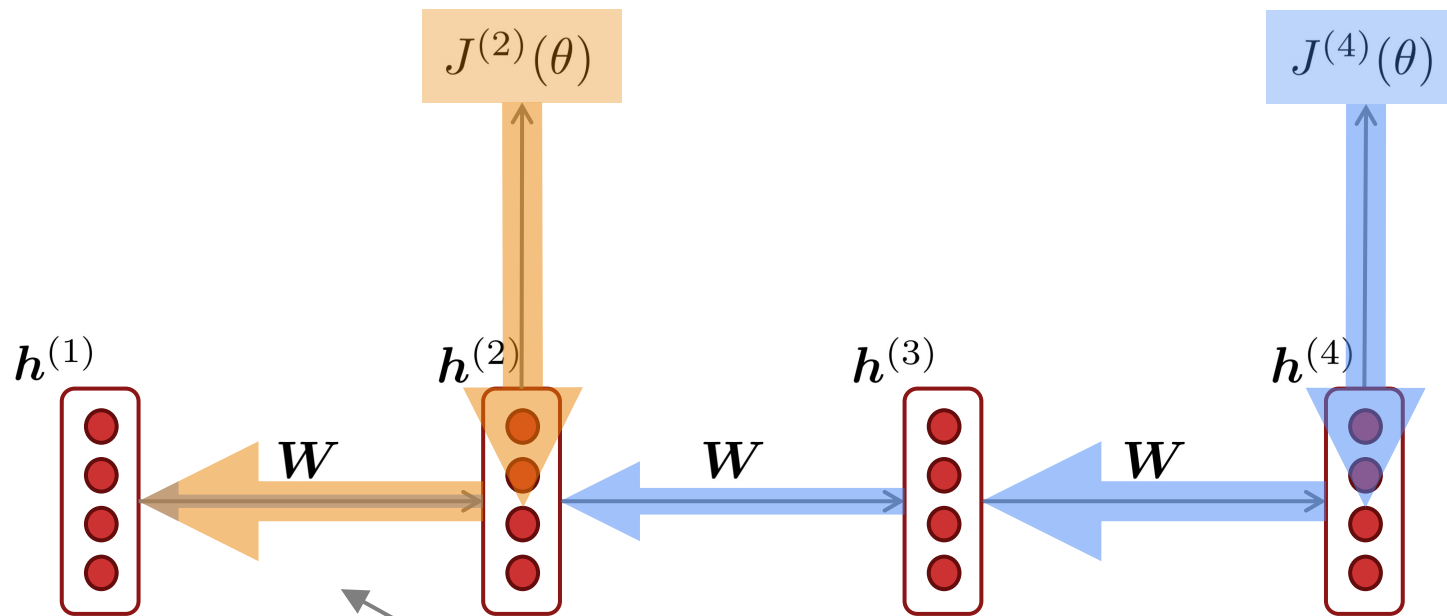
- What's wrong with  $W_h^\ell$  ?
- Consider if the eigenvalues of  $W_h$  are all less than 1:  
 $\lambda_1, \lambda_2, \dots, \lambda_n < 1$   
 $q_1, q_2, \dots, q_n$  (eigenvectors)
- We can write  $\frac{\partial J^{(i)}(\theta)}{\partial h^{(i)}} W_h^\ell$  using the eigenvectors of  $W_h$  as a basis:

$$\frac{\partial J^{(i)}(\theta)}{\partial h^{(i)}} W_h^\ell = \sum_{i=1}^n c_i \lambda_i^\ell q_i \approx \mathbf{0} \text{ (for large } \ell \text{)}$$

Approaches 0 as  $\ell$  grows, so gradient vanishes

- What about nonlinear activations  $\sigma$  (i.e., what we use?)
  - Pretty much the same thing, except the proof requires  $\lambda_i < \gamma$  for some  $\gamma$  dependent on dimensionality and  $\sigma$

# Why is vanishing gradient a problem?



Gradient signal from far away is lost because it's much smaller than gradient signal from close-by.

So, model weights are updated only with respect to near effects, not long-term effects.

# Effect of vanishing gradient on RNN-LM

- **LM task:** *When she tried to print her tickets, she found that the printer was out of toner. She went to the stationery store to buy more toner. It was very overpriced. After installing the toner into the printer, she finally printed her \_\_\_\_\_*
- To learn from this training example, the RNN-LM needs to **model the dependency** between “*tickets*” on the 7<sup>th</sup> step and the target word “*tickets*” at the end.
- But if the gradient is small, the model **can't learn this dependency**
  - So, the model is **unable to predict similar long-distance dependencies** at test time

# Why is exploding gradient a problem?

- If the gradient becomes too big, then the SGD update step becomes too big:

$$\theta^{new} = \theta^{old} - \overbrace{\alpha}^{\text{learning rate}} \underbrace{\nabla_{\theta} J(\theta)}_{\text{gradient}}$$

- This can cause **bad updates**: we take too large a step and reach a weird and bad parameter configuration (with large loss)
  - You think you've found a hill to climb, but suddenly you're in Iowa
- In the worst case, this will result in **Inf** or **NaN** in your network (then you have to restart training from an earlier checkpoint)



# Gradient clipping: solution for exploding gradient

- **Gradient clipping**: if the norm of the gradient is greater than some threshold, scale it down before applying SGD update

---

**Algorithm 1** Pseudo-code for norm clipping

---

```
 $\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$   
if  $\|\hat{\mathbf{g}}\| \geq threshold$  then  
     $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$   
end if
```

---

- **Intuition**: take a step in the same direction, but a smaller step
- In practice, **remembering to clip gradients is important**, but exploding gradients are an easy problem to solve

# How to fix the vanishing gradient problem?

- The main problem is that *it's too difficult for the RNN to learn to preserve information over many timesteps.*

- In a vanilla RNN, the hidden state is constantly being rewritten

$$\mathbf{h}^{(t)} = \sigma \left( \mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_x \mathbf{x}^{(t)} + \mathbf{b} \right)$$

- First off next time: How about an RNN with separate memory which is added to?
  - LSTMs
- And then: Creating more direct and linear pass-through connections in model
  - Attention, residual connections, etc.

## 5. Recap

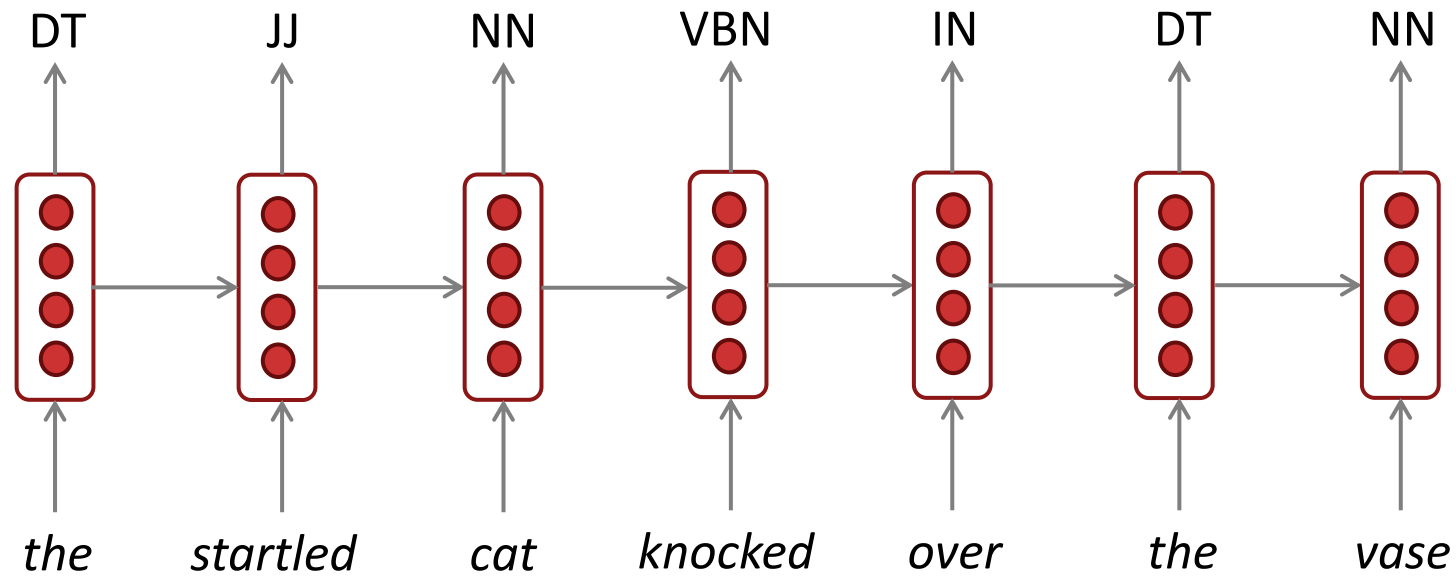
- **Language Model**: A system that predicts the next word
- **Recurrent Neural Network**: A family of neural networks that:
  - Take sequential input of any length
  - Apply the same weights on each step
  - Can optionally produce output on each step
- Recurrent Neural Network  $\neq$  Language Model
- We've shown that RNNs are a great way to build a LM (despite some problems)
- RNNs are also useful for much more!

# Why should we care about Language Modeling?

- Language Modeling is a **benchmark task** that helps us **measure our progress** on predicting language use
- Language Modeling is a **subcomponent** of many NLP tasks, especially those involving **generating text** or **estimating the probability of text**:
  - Predictive typing
  - Speech recognition
  - Handwriting recognition
  - Spelling/grammar correction
  - Authorship identification
  - Machine translation
  - Summarization
  - Dialogue
  - etc.
- Everything else in NLP has now been rebuilt upon Language Modeling: **GPT-3 is an LM!**

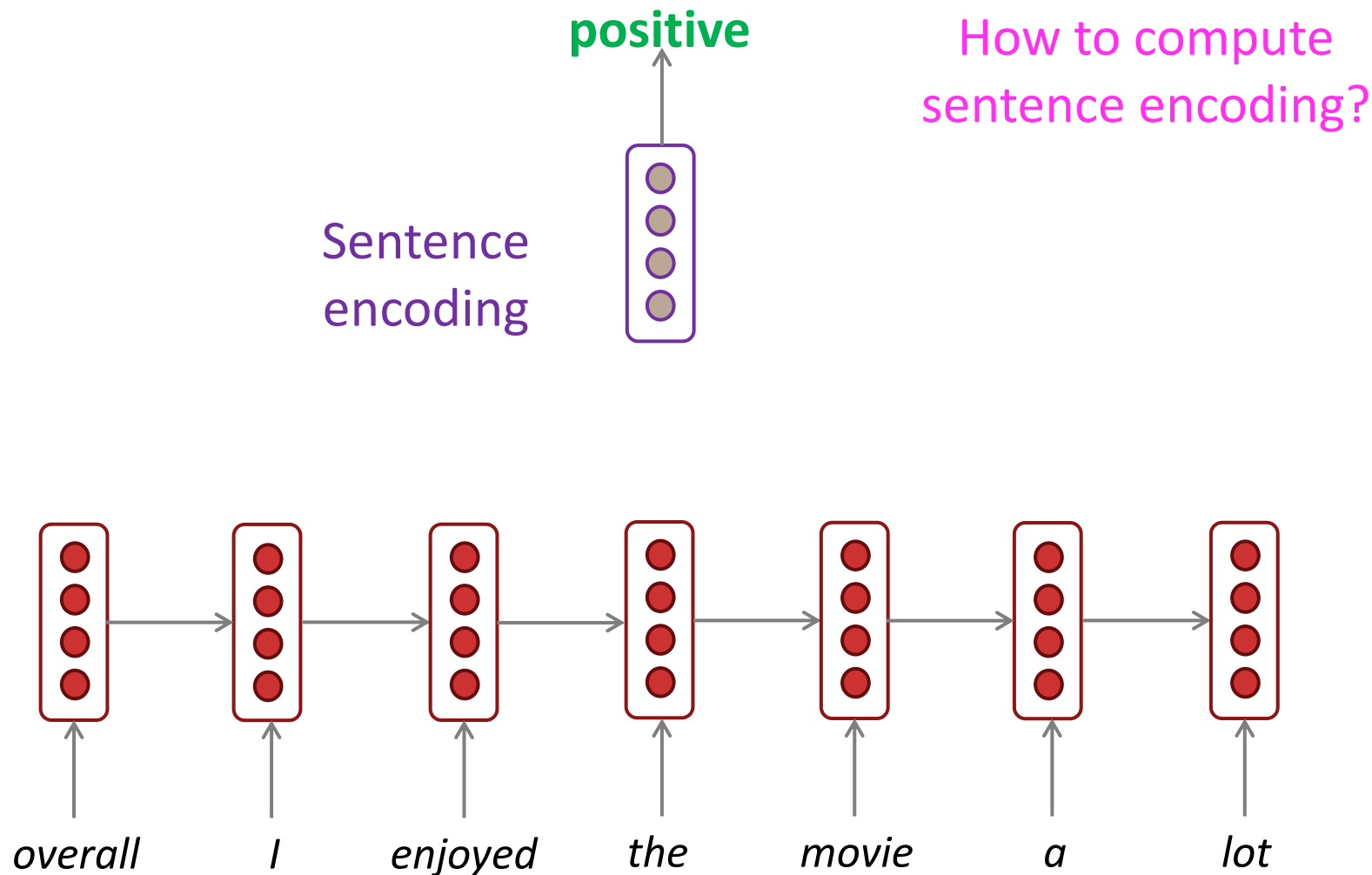
## Other RNN uses: RNNs can be used for sequence tagging

e.g., part-of-speech tagging, named entity recognition



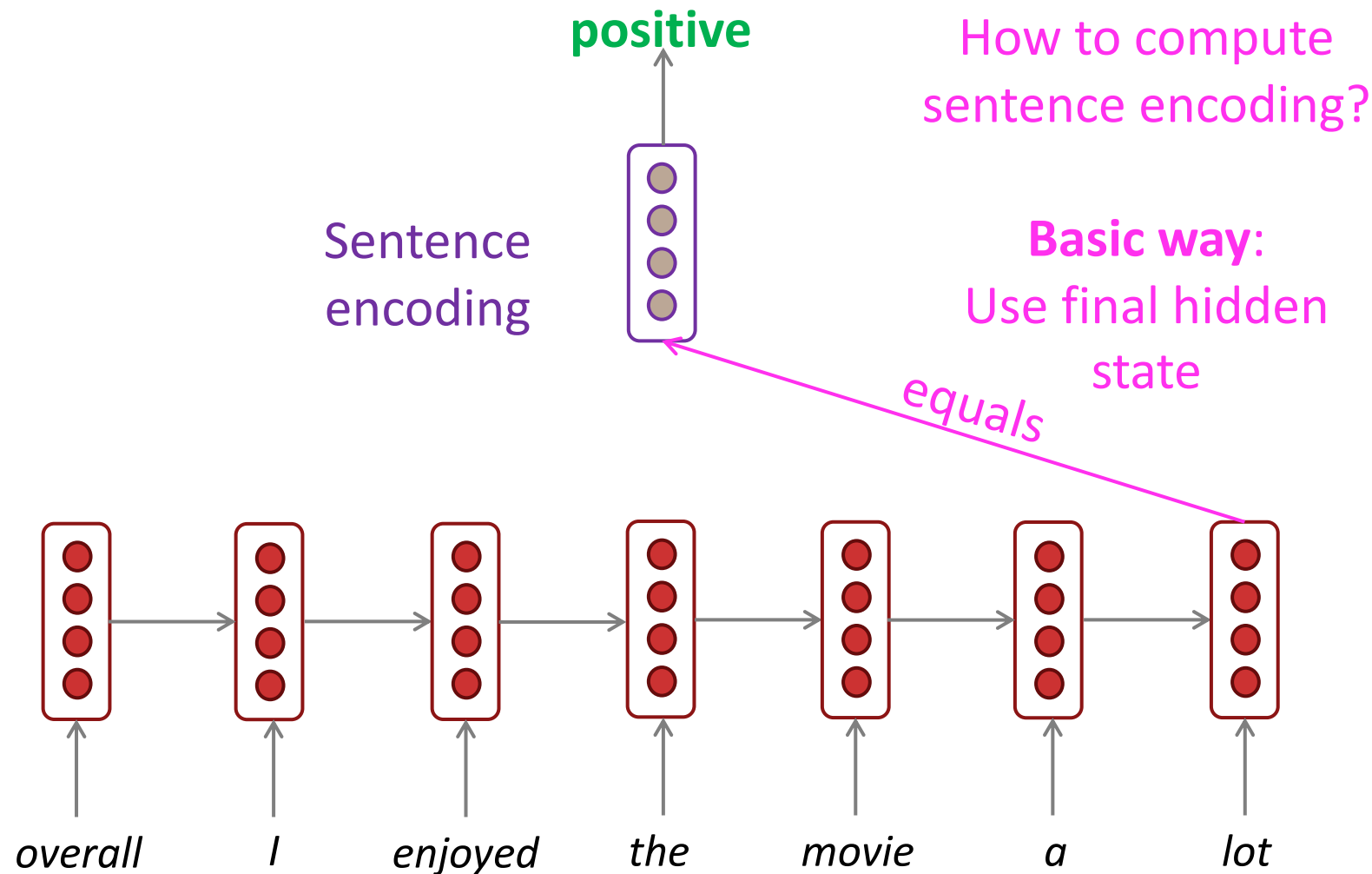
# RNNs can be used for sentence classification

e.g., sentiment classification



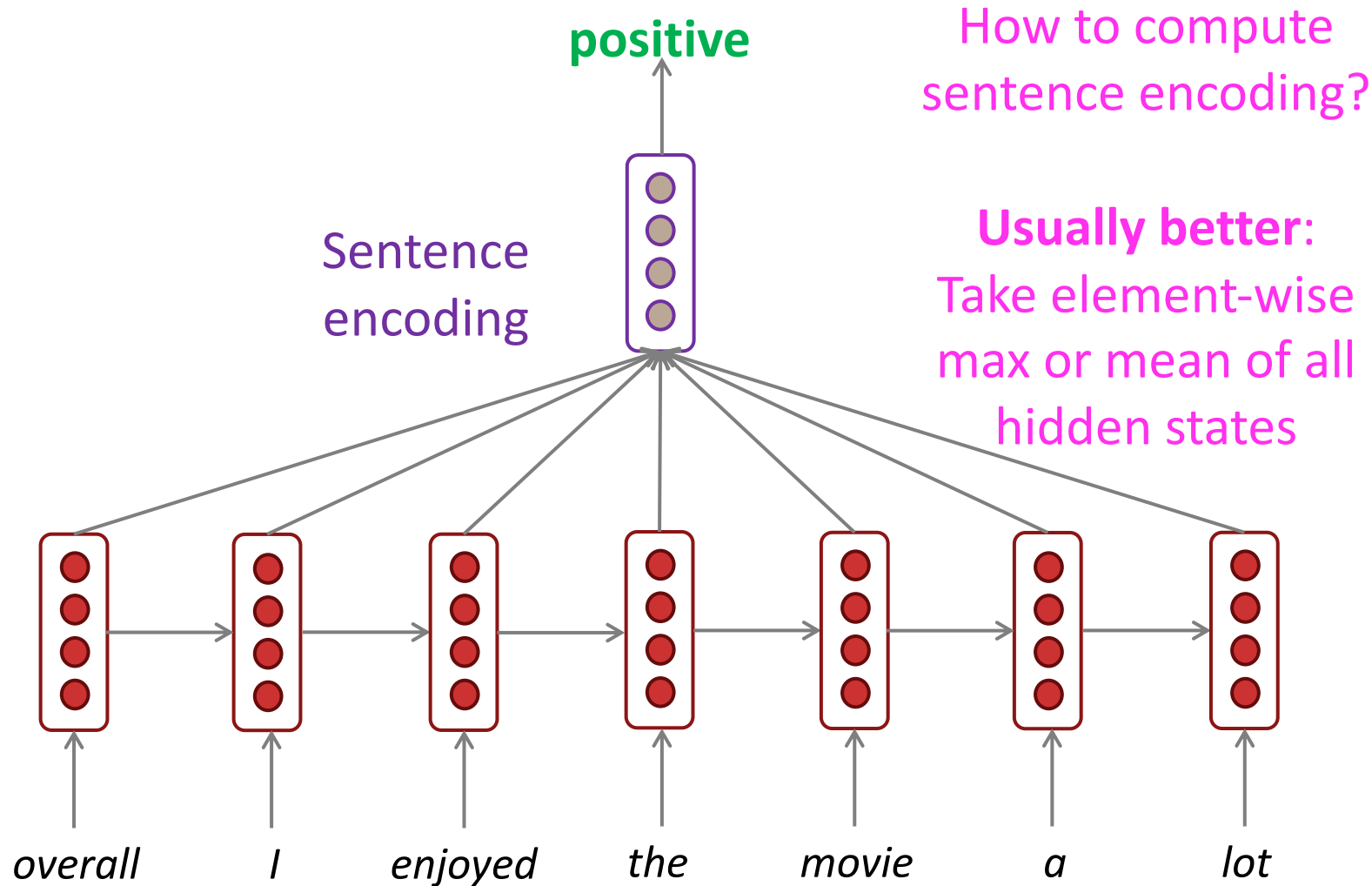
# RNNs can be used for sentence classification

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# RNNs can be used for sentence classification

e.g., sentiment classification

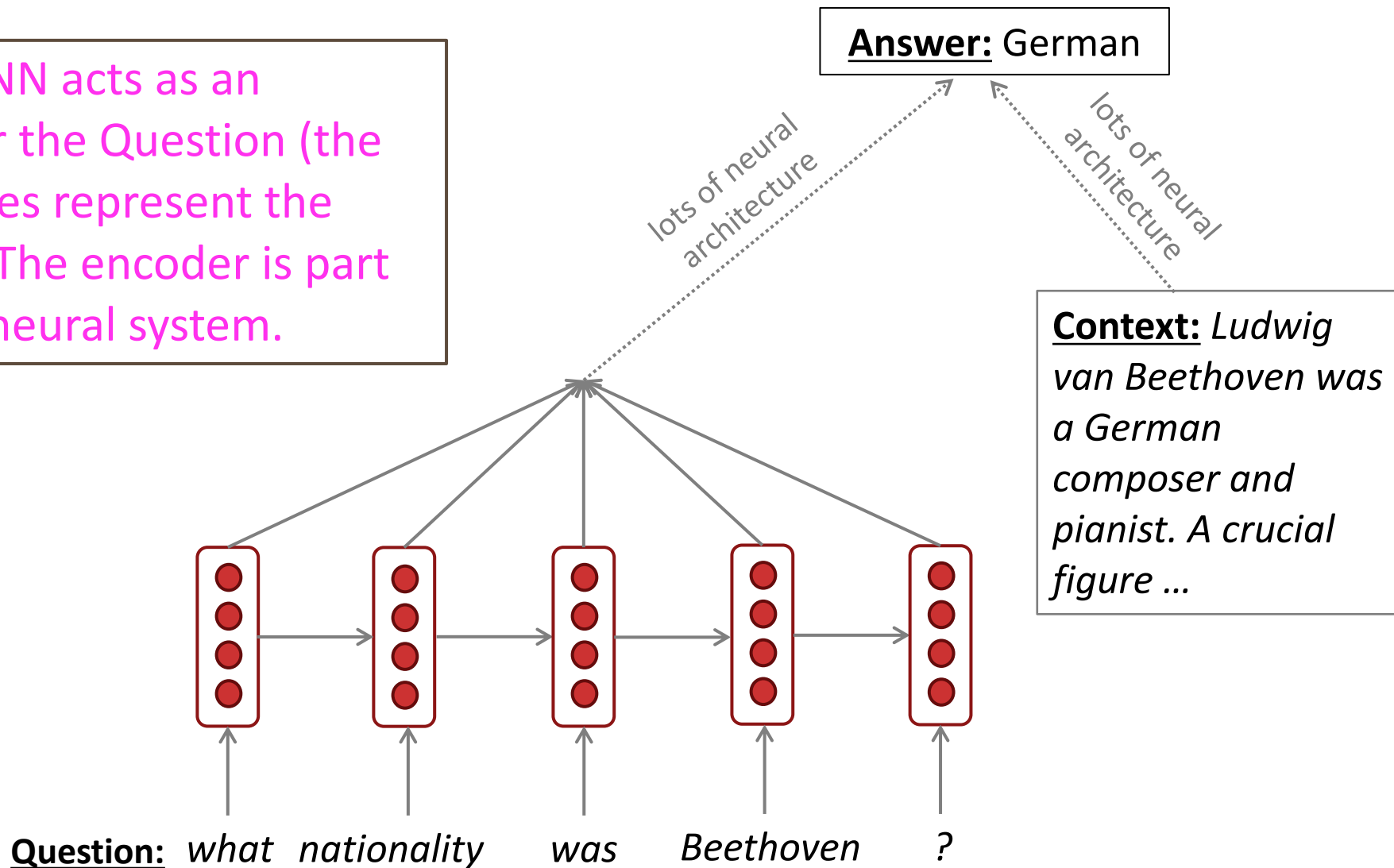




# RNNs can be used as an encoder module

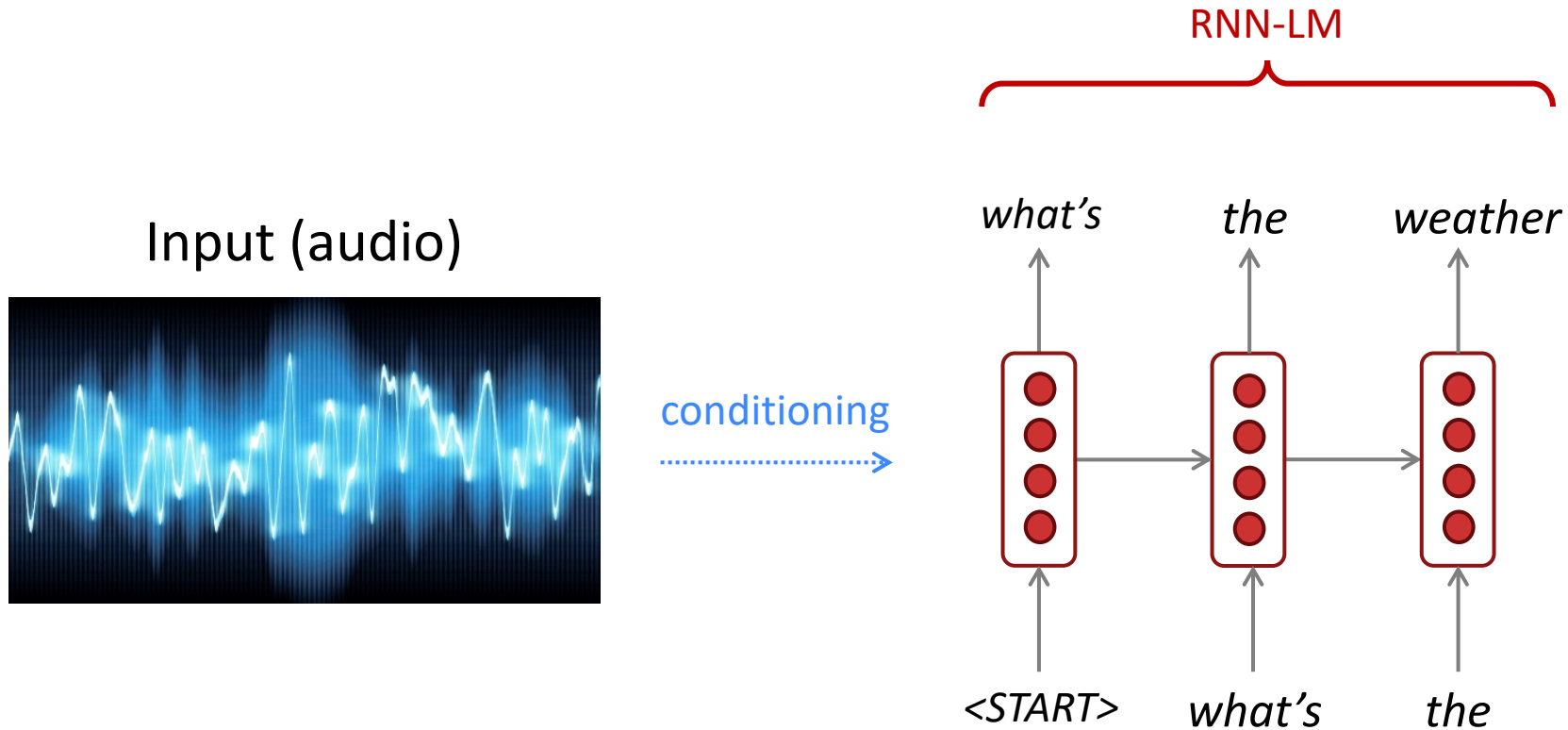
e.g., question answering, machine translation, *many other tasks!*

Here the RNN acts as an **encoder** for the Question (the hidden states represent the Question). The encoder is part of a larger neural system.



# RNN-LMs can be used to generate text

e.g., speech recognition, machine translation, summarization



This is an example of a *conditional language model*.  
We'll see Machine Translation in much more detail starting next lecture.

# Terminology and a look forward

The RNN described in this lecture = **simple**/vanilla/**Elman** RNN



**Next lecture:** You will learn about other RNN flavors

like **LSTM**



and **GRU**



and multi-layer RNNs



**By the end of the course:** You will understand phrases like  
*“stacked bidirectional LSTMs with residual connections and self-attention”*

