Predicting Structures: Conditional Models and Local Classifiers

CS 6355: Structured Prediction

THE UNIVERSITY OF UTAH

Outline

- Sequence models
- Hidden Markov models
  - Inference with HMM
  - Learning
- Conditional Models and Local Classifiers
- Global models
  - Conditional Random Fields
  - Structured Perceptron for sequences

Today’s Agenda

- Conditional models for predicting sequences

- Maximum Entropy Markov Models

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HMM redux

- The independence assumption

$$P(x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_n) = P(y_1) \prod_{i=1}^{n-1} P(\, y_{i+1} \mid y_i \,) \prod_{i=1}^{n} P(\, x_i \mid y_i \,)$$

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- Training via maximum likelihood

$$\max_{\pi, A, B} P(D \mid \pi, A, B) = \max_{\pi, A, B} \prod_{i} P(\mathbf{x}_i, \mathbf{y}_i \mid \pi, A, B)$$

We are optimizing joint likelihood of the input and the output for training

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At prediction time, we only care about the probability of output given the input:

$$P(y_1, y_2, \cdots, y_n \mid x_1, x_2, \cdots, x_n)$$

Why not directly optimize this *conditional likelihood* instead?

Modeling next-state directly

- Instead of modeling the joint distribution $P(\mathbf{x}, \mathbf{y})$, focus on $P(\mathbf{y} \mid \mathbf{x})$ only
  - Which is what we care about eventually anyway

    (At least in this context)

- For sequences, different formulations
  - Maximum Entropy Markov Model [McCallum, et al 2000]

  (other names: discriminative/conditional Markov model, projection-based Markov model…)

Generative vs Discriminative models

- Generative models
  - learn P(x, y)
  - Characterize how the data is generated (both inputs and outputs)
  - Eg: Naïve Bayes, Hidden Markov Model

- Discriminative models
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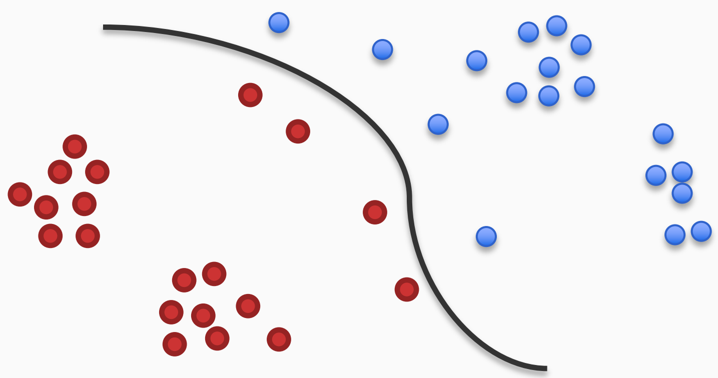

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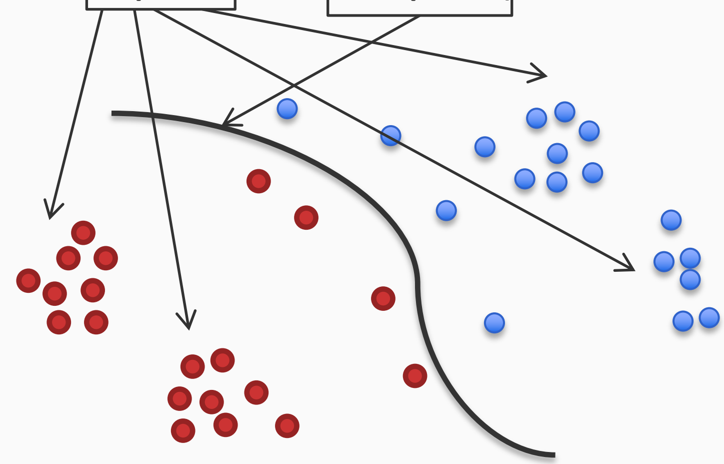

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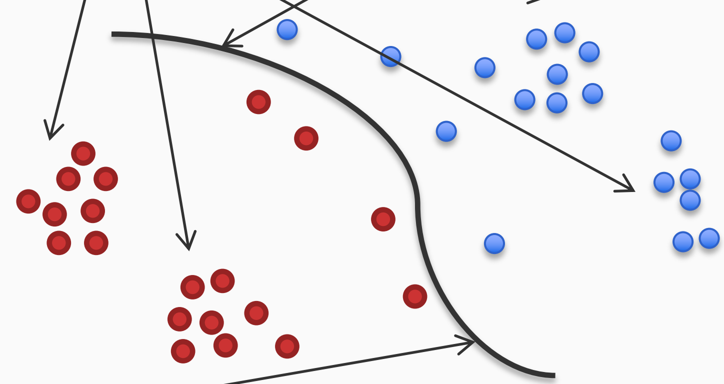

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HMM redux

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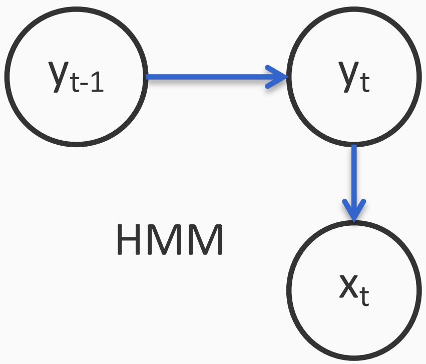
Let's revisit the independence assumptions

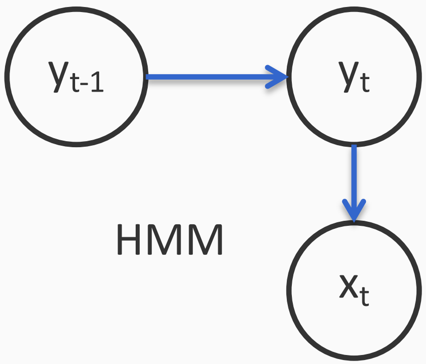


$$P(\, y_i \mid y_{i-1}, \text{anything else} \,) = P(\, y_i \mid y_{i-1} \,)$$

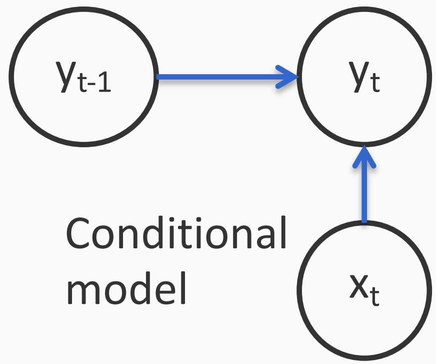
$$P(\, x_i \mid y_i, \text{anything else}\,) = P(\, x_i \mid y_i \,)$$

Another independence assumption

$$P(y_i \mid y_{i-1}, y_{i-2}, \cdots, x_i, x_{i-1}, \cdots) = P(y_i \mid y_{i-1}, x_i)$$



HMM



Conditional model

Another independence assumption

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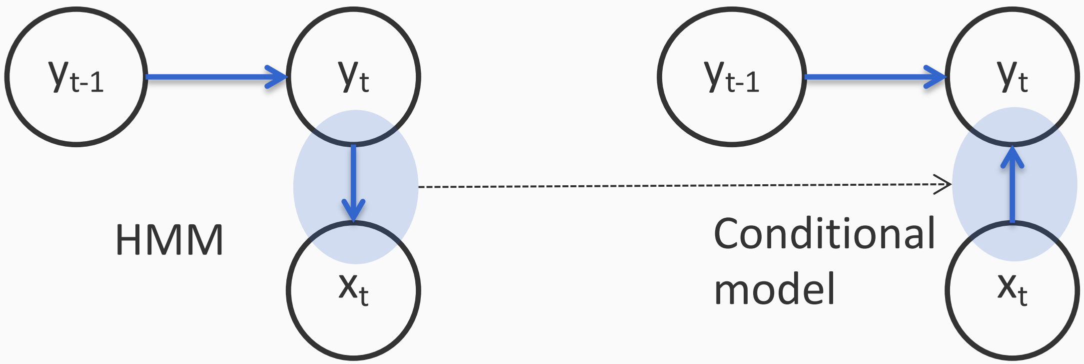

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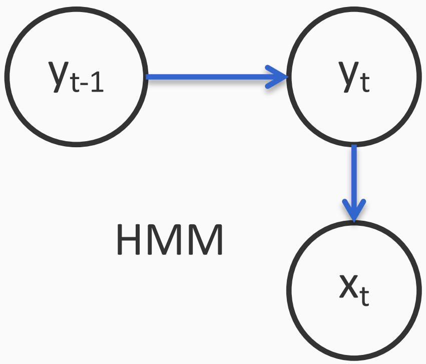

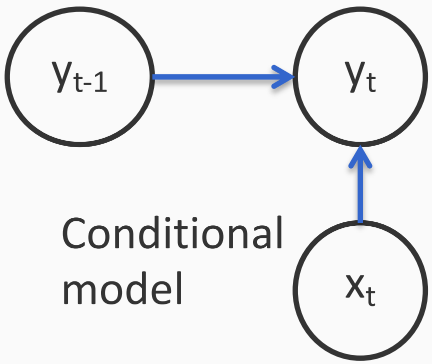

This assumption lets us write the conditional probability of the entire output sequence $\mathbf{y}$ as

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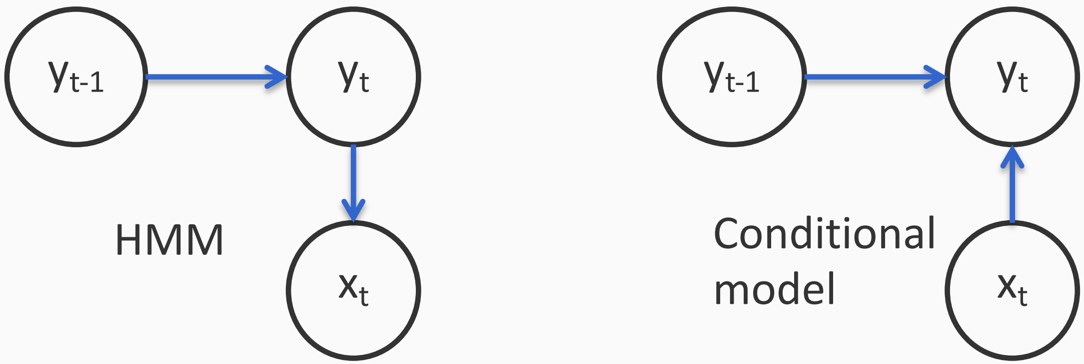

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We need to learn this function

Modeling $P(y_i \mid y_{i-1}, x_i)$

This is a multiclass classifier whose input is the pair y_{i-1}, x_i, and the label is the state y_i

Different approaches possible

1. Train a *maximum entropy* classifier (i.e., a multiclass logistic regression classifier)

2. Or, ignore the fact that we are predicting a probability, we only care about maximizing some *score.* Train any multiclass classifier, using say the perceptron algorithm

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In any case, we can use any features from y_{i-1} and x_i. This was not possible with HMMs.

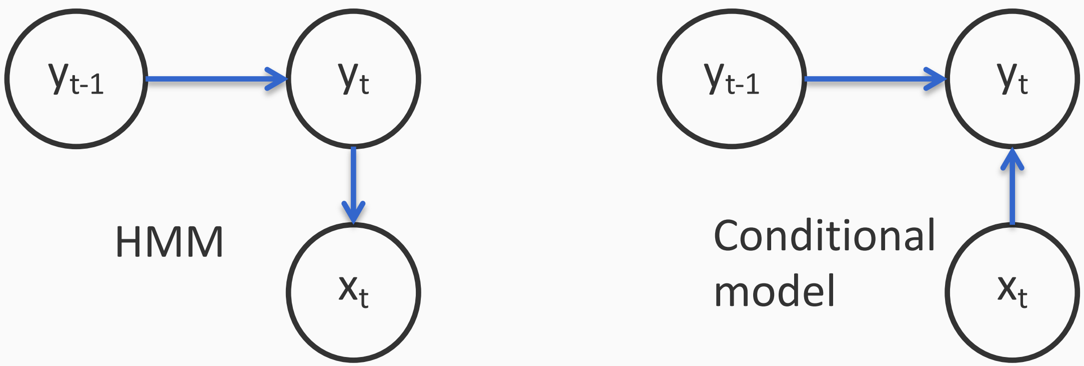
Today’s Agenda

- Conditional models for predicting sequences

- Maximum Entropy Markov Models

The next-state model

$$P(y_i \mid y_{i-1}, y_{i-2}, \cdots, x_i, x_{i-1}, \cdots) = P(y_i \mid y_{i-1}, x_i)$$



This assumption lets us write the conditional probability of the output as

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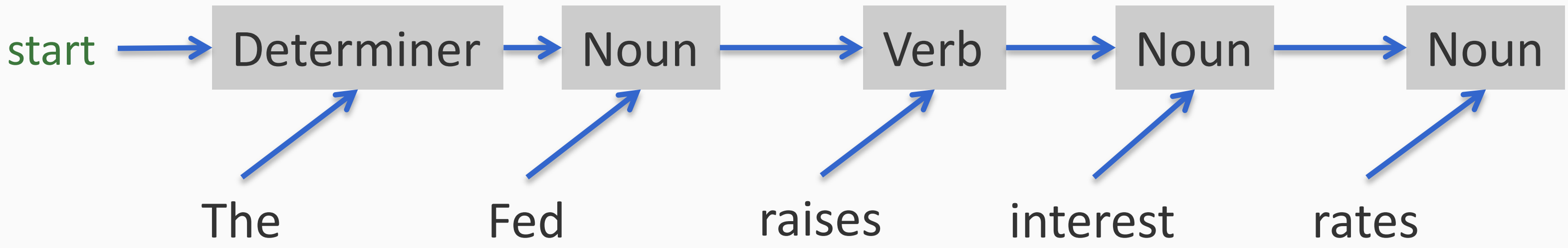
2. Or, ignore the fact that we are predicting a probability, we only care about maximizing some *score.* Train any multiclass classifier, using say the perceptron algorithm

For both cases*:*

- Use rich features that depend on input and previous state
- We can increase the dependency to arbitrary neighboring x_i's
  - Eg. Neighboring words influence this words POS tag

Maximum Entropy Markov Model

Goal: Compute P(**y** | **x**) $$P(y_i|y_{i-1}, \mathbf{x}) \propto \exp\left(\mathbf{w}^T \phi(\mathbf{x}, i, y_i, y_{i-1})\right)$$



The prediction task: Using the entire input and the current label, predict the next label

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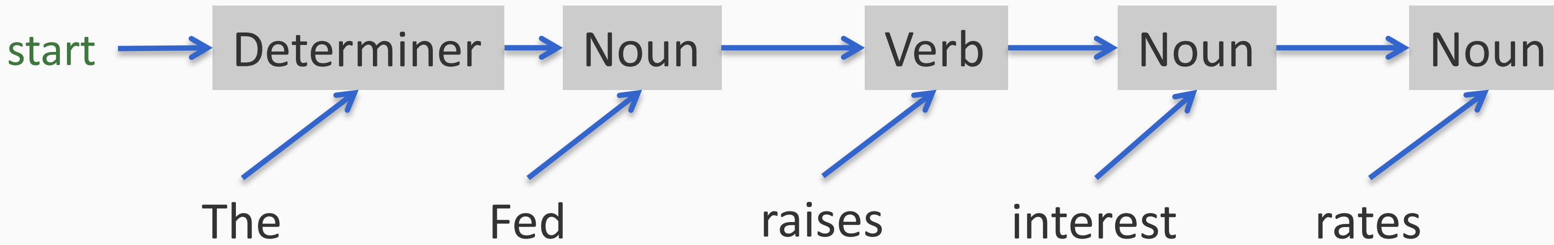

word

Caps

-es

Previous

To model the probability, first, we need to define features for the current classification problem

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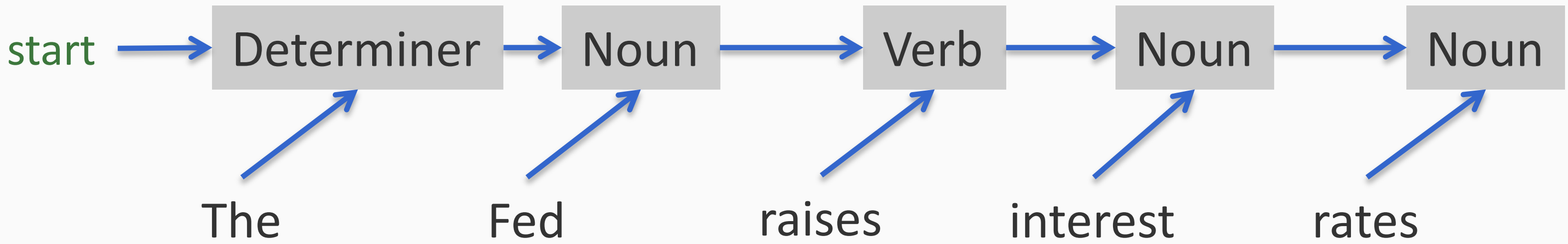

$$\begin{array}{ll} \textit{word} & \\ \textit{Caps} & \\ \textit{-es?} & \\ \textit{Previous} & \end{array} \begin{bmatrix} \text{The} \\ \text{Y} \\ \text{N} \\ \text{start} \end{bmatrix}$$

$\phi(\mathbf{x}, 0, \text{start})$

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start → Determiner → Noun → Verb → Noun → Noun

The → Determiner; Fed → Noun; raises → Verb; interest → Noun; rates → Noun

	$\phi(\mathbf{x}, 0, \text{start})$	$\phi(\mathbf{x}, 1, \text{Det.})$
word	The	Fed
Caps	Y	Y
-es?	N	N
Previous	start	Determiner

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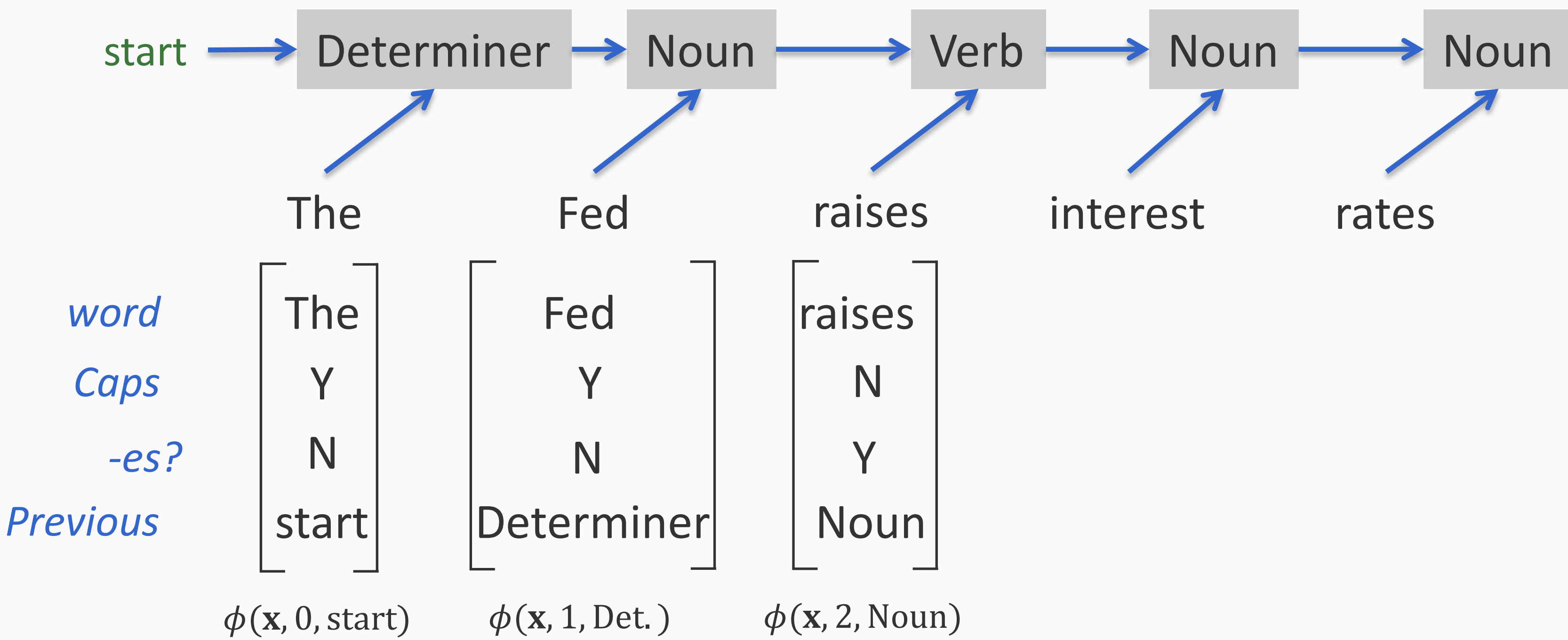

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The Fed raises interest rates

	$\phi(\mathbf{x},0,\text{start})$	$\phi(\mathbf{x},1,\text{Det.})$	$\phi(\mathbf{x},2,\text{Noun})$	$\phi(\mathbf{x},3,\text{Verb})$
word	The	Fed	raises	interest
Caps	Y	Y	N	N
-es?	N	N	Y	N
Previous	start	Determiner	Noun	Verb

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Can get very creative here

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The ↗ Determiner, Fed ↗ Noun, raises ↗ Verb, interest ↗ Noun, rates ↗ Noun

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Caps	Y	Y	N	N	N
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Previous	start	Determiner	Noun	Verb	Noun

Can get very creative here

Compare to HMM: Only depends on the word and the previous tag

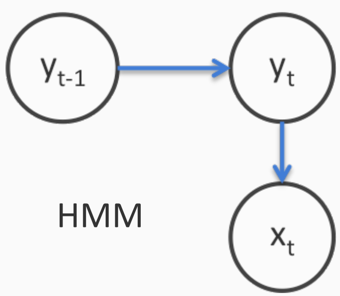
start → Determiner → Noun → Verb → Noun → Noun

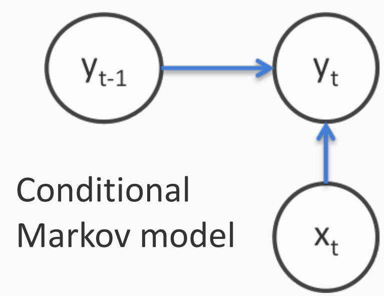
Determiner ↓ The, Noun ↓ Fed, Verb ↓ raises, Noun ↓ interest, Noun ↓ rates

Questions?

Using MEMM

- Training
  - Next-state predictor **locally** as maximum likelihood
    - Similar to any maximum entropy classifier
- Prediction/decoding
  - Modify the Viterbi algorithm for the new independence assumptions





$$\text{score}_i(s) = \max_{y_{i-1}} P(s|y_{i-1})P(x_i|s)\text{score}_{i-1}(y_{i-1})$$

$$\text{score}_i(s) = \max_{y_{i-1}} P(s|y_{i-1}, x_i)\text{score}_{i-1}(y_{i-1})$$

Generalization: Any multiclass classifier

- **Viterbi decoding**: we only need a score for each decision
  - So far, probabilistic classifiers

- In general, use any learning algorithm to build get a score for the label y_i given y_{i-1} and $\mathbf{x}$
  - Multiclass versions of perceptron, SVM
  - Just like MEMM, these allow arbitrary features to be defined

Exercise: Viterbi needs to be re-defined to work with sum of scores rather than the product of probabilities

Comparison to HMM

What we gain

1. Rich feature representation for inputs
    - Helps generalize better by thinking about properties of the input tokens rather than the entire tokens
    - Eg: If a word ends with –es, it might be a present tense verb (such as raises). Could be a feature; HMM cannot capture this

2. Discriminative predictor
    - Model P(**y** | **x**) rather than P(**y**, x)
    - *Joint* vs *conditional*

Questions?

Outline

- Conditional models for predicting sequences
- Log-linear models for multiclass classification
- Maximum Entropy Markov Models
  - The Label Bias Problem

The next-state model for sequences

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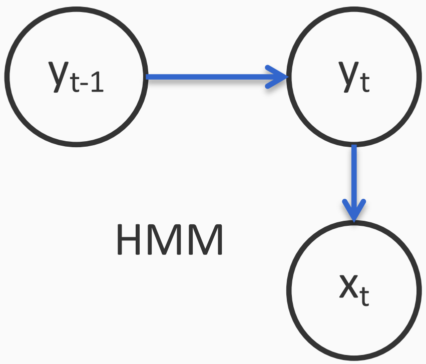

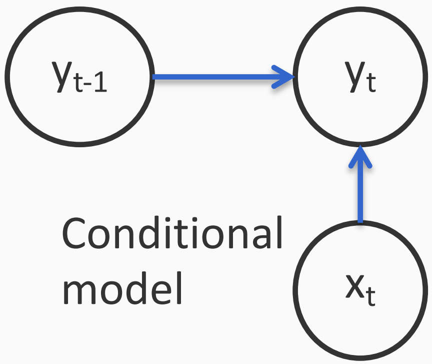

This assumption lets us write the conditional probability of the output as

$$P(\mathbf{y}|\mathbf{x}) = \prod_i P(y_i|y_{i-1}, x_i)$$

We need to train local multiclass classifiers that predicts the next state given the previous state and the input

…local classifiers → Label bias problem

Let's look at the independence assumption

$$P(y_i|y_{i-1}, y_{i-2}, \cdots, x_i, x_{i-1}, \cdots) = P(y_i|y_{i-1}, x_i)$$

"Next-state" classifiers are locally normalized

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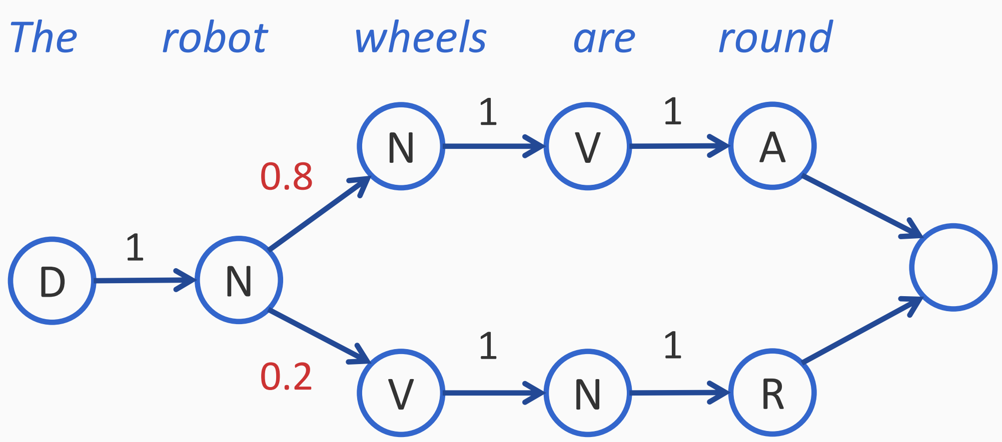
Eg: Part-of-speech tagging the sentence



Suppose these are the only state transitions allowed

Example based on [Wallach 2002]

…local classifiers → Label bias problem

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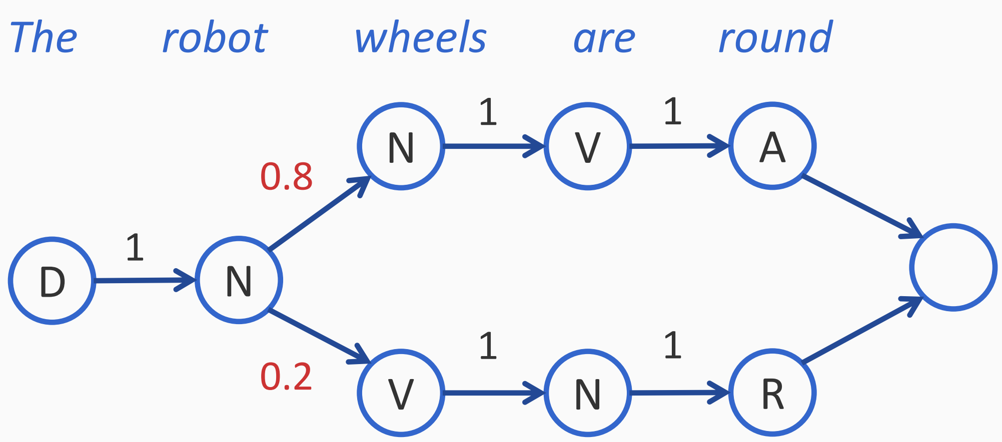

Option 1: P(D | The) ×
P(N | D, robot) ×
P(N | N, wheels) ×
P(V | N, are) ×
P(A | V, round)

Option 2: P(D | The) ×
P(N | D, robot) ×
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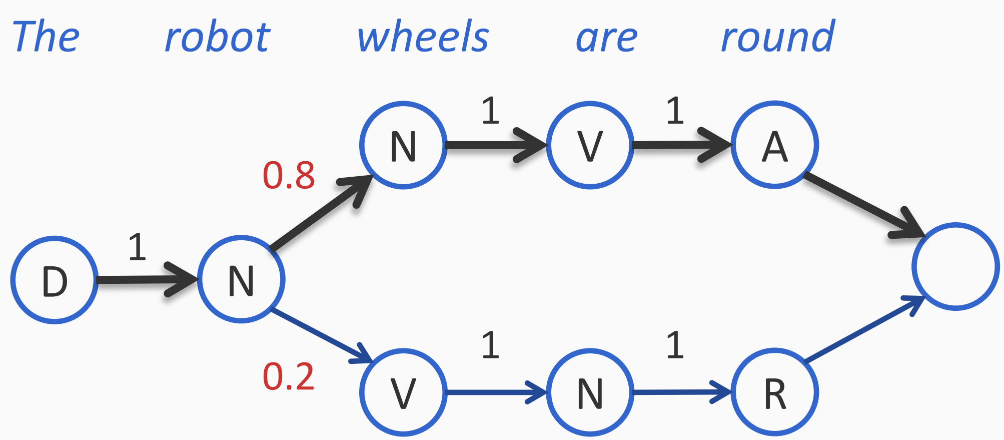

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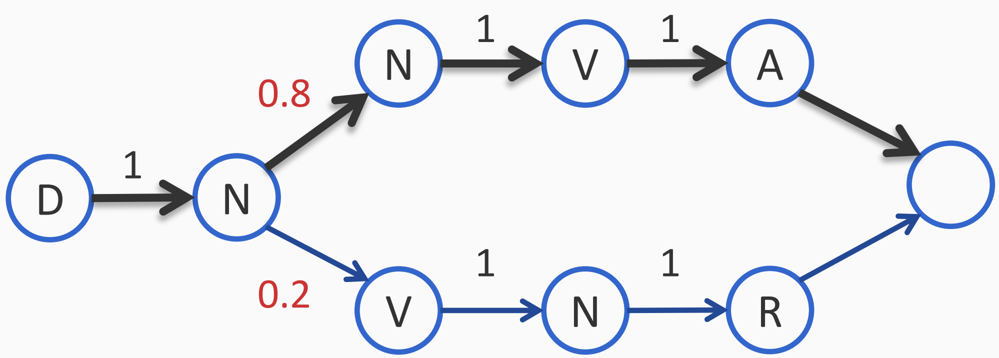
P(N | N, wheels) ×
P(V | N, are) ×
P(A | V, round)

Option 2: P(D | The) ×
P(N | D, robot) ×
P(V | N, wheels) ×
P(N | V, are) ×
P(R| N, round)

…local classifiers → Label bias problem

The *robot* *wheels* *are* *round*



Option 1: P(D | The) ×
P(N | D, robot) ×
P(N | N, wheels) ×
~~P(V | N, are)~~ × P(V | N, Fred) ×
P(A | V, round)

Option 2: P(D | The) ×
P(N | D, robot) ×
P(V | N, wheels) ×
~~P(N | V, are)~~ × P(N | V, Fred) ×
P(R| N, round)

Suppose these are the only state transitions allowed

The *robot* *wheels* *Fred* *round*

The path scores are the same

Even if the word Fred is never observed as a verb in the data, it will be predicted as one

The input *Fred* does not influence the output at all

Label Bias

- States with a single outgoing transition effectively ignore their input
  - States with lower-entropy next states are less influenced by observations
- Why?
  - Because each the next-state classifiers are locally normalized
  - If a state has fewer next states, each of those will get a higher probability mass
    - …and hence preferred
- Side note: Surprisingly doesn't affect some tasks
  - Eg: part-of-speech tagging

Summary: Local models for Sequences

- Conditional models
- Use rich features in the mode
- Possibly suffer from label bias problem

(Other “local” models may have their own version of the label bias problem too.)

Outline

- Sequence models
- Hidden Markov models
  - Inference with HMM
  - Learning
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  - Conditional Random Fields
  - Structured Perceptron for sequences

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